Poletechnic Lecture Note

# FLUID MECHANICS

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**The centre of gravity (G) and moment of inertia (I) of some important geometrical figures:**

| S.No. | Name of figure | C.G. from the base | Area | I about an axis passing through C.G. and parallel to the base | I about base |
|---|---|---|---|---|---|
| 1. | Triangle Fig. 3.3 | $x = \frac{h}{3}$ | $\frac{bh}{2}$ | $\frac{bh^3}{36}$ | $\frac{bh^3}{12}$ |
| 2. | Rectangle Fig. 3.4 | $x = \frac{d}{2}$ | $bd$ | $\frac{bd^3}{12}$ | $\frac{bd^3}{3}$ |
| 3. | Circle Fig. 3.5 | $x = \frac{d}{2}$ | $\frac{\pi d^2}{4}$ | $\frac{\pi d^4}{64}$ | — |
| 4. | Trapezium Fig. 3.6 | $x = \left[\frac{2a+b}{a+b}\right]\frac{h}{3}$ | $\left(\frac{a+b}{2}\right)h$ | $\left(\frac{a^2+4ab+b^2}{36(a+b)}\right) \times h^2$ | — |

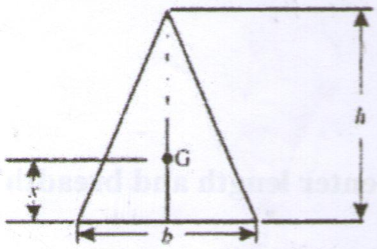


Fig. 3.3

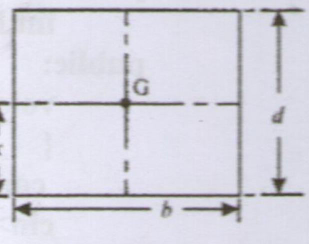


Fig. 3.4

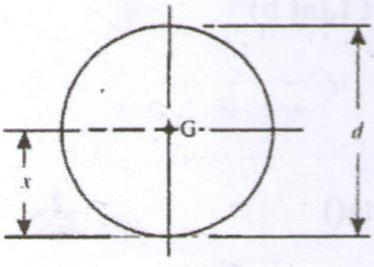


Fig. 3.5

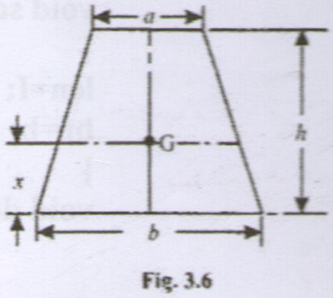


Fig. 3.6

Poletechnic Lecture Note

Fluid Mechanics

Subject chapter 2: Fluid properties Date No.

| Basic Unit | SI (system International units) | BG (Britsh system) |
|---|---|---|
| Mass | Kilogram ⇒ Kg | slug |
| Length | meter ⇒ m | Foot ⇒ ft |
| Time | Second ⇒ S | second ⇒ sec |
| Temprature | Ⓐ Absolute ⇒ kelvin ⇒ °K | Rankine ⇒ °R |
| | Ⓑ ordinary ⇒ celsius ⇒ °C | Fahrenheit ⇒ °F |

* property :- any characteristic can be measured or observed

أي سمة يمكن قياسها أو ملاحظتها

* property ⟶ Extensive ⟶ energy    شاملة : تعتمد على ال mass

⟶ Intensive ⟶ Temprature    مركزة : لا تعتمد على ال mass

* $\underset{\Downarrow \text{ intensive property}}{\text{specific property}} = \dfrac{\text{extensive property}}{\text{mass}}$

* specific volume $(v) = \dfrac{\forall}{m} = \dfrac{1}{\rho}$ [$m^3/kg$]    density $\rho$

حجم لكل وحدة كتلة    $\rho = \dfrac{m}{\forall} = \dfrac{kg}{m^3}$

* Fluid mechanics ⟶ liquid

⟶ gas

* phase ⟶ solid

⟶ liquid

⟶ vapor

Subject Date No.

* compersible fluid (مائع قابل للإنضغاط (تطلق على gas) $\Longrightarrow$ important change in volume

* incompersible fluid (مائع غير قابل للإنضغاط (تطلق على liquid) $\Longrightarrow$ not important change in volume

incompersible $\Longrightarrow$ $\rho$ = constant

** specific weight :-

$$\text{weight} = w = mg$$

$$\text{specific weight fluid} = \frac{mg}{\forall} = \frac{\rho \forall g}{\forall} = \rho g = \gamma \; [N/m^3]$$

** specific gravity :- (s.g) $\Rightarrow$ The ratio of the specific weight of a fluid to weight of water at the stand condition.

$$s.g = \frac{\gamma \text{ Fluid}}{\gamma \text{ water}} = \frac{(\rho g)_F}{(\rho g)_w} = \frac{\rho_F}{\rho_w}$$

$\gamma_{H_2O} = 1$ at $4\,^\circ C$ , $\rho_{H_2O} = 1000 \; kg/m^3$

$(s.g)_w = 1$ , $(s.g)_{Hg} = 13.6$

$$\gamma_F = (s.g)_F \cdot \gamma_w$$

$$\rho_F = (s.g)_F \; \rho_w$$

N O T E B O O K

Subject equation of state gas Date No.

* Ideal gas Law :-

state ⇒ discribed material both property to known other property

state ⇒ T , P , V

$$\boxed{PV = mRT}$$

P: Pressure → Kpa

V: volume → $m^3$

m: mass → Kg

T: Temperature → k

$°k = °C + 273.15$

R: particular gas constant ⇒ kJ/ Kg.°k

R for air = 0.287 [kJ/kg.°k]

P — R

Pa → J

Kpa → kJ

$$P = \frac{m}{v} RT \quad , \quad \rho = \frac{m}{v}$$

$$P = \rho RT \Rightarrow \rho = \frac{P}{RT}$$

For only gas

ex:-

air

m = ?? ✓

diamention of room 4 m * 5 m * 6

25 c

100 kpa

density = ?? ✓

sol:-

$PV = mRT$

$m = \frac{PV}{RT}$ , $V = 4*5*6 = 120 [m^3]$

$m = \frac{100 * 120}{0.287 * [25+273]} = 140.3 \text{ kg}$

$\rho = \frac{P}{RT} = \frac{100}{0.287 * 298.15} = 1.17 [kg/m^3]$

Subject Date No.

* example 2,1 :- Air at standard sea-level ($p = 101\ kN/m^2$) has a temperature of 4 °c . what is the density of the air?

| | |
|---|---|
| $P_{Air} = 101\ [kN/m^2]$ | $\rho = \frac{P}{RT} = \frac{101}{0,287 * [4+273,15]}$ |
| $T_{Air} = 4°c$ | |
| $R_{Air} = 0,287\ [kJ/kg.K]$ | $\rho = 1,27\ kg/m^3$ |

* problem 2,4 :- An engineer living at an elevation of 800m is conducting experiments to verify predictions of glider performance. To process data , density of ambient air is needed. The engineer measure tempreture (296,~~5~~ K) and atmospheric pressure (92,45 kpa). calculate density in unit of $kg/m^3$ . compare the calculated value with data from table A3 and make recommendation about the effects of elevation on density?

$T = 74,3\ °F \Rightarrow 74,3 = 1,8 * T_{°C} + 32$

$T = 23,~~5~~\ °C = 296,5\ [K]$

$P = 92,45\ [kpa]$

$R_{Air} = 0,287\ [kJ/kg.K]$

$\rho = 1,22\ [kg/m^3]$

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ⓐ calculate density in unit $kg/m^3$?

$$\rho = \frac{P}{RT} = \frac{92{,}45}{0{,}287 * 296{,}5} = 1{,}086\ [kg/m^3]$$

ⓑ $\rho$ in Table A3 value:-

$$\rho = 1{,}22\ kg/m^3$$

Proplem 2.5:- Calculate the density and specific weight of carbon dioxide at pressure of 300 $kN/m^2$ absolute and 60 °C?

$P_{CO_2} = 300\ kN/m^2$

$T = 60°C = 333°K$

$R_{CO_2} = 189\ J/kg.k = 0{,}189\ [kJ/kg\,°k]$

① density $\Longrightarrow$ $\rho = \frac{P}{RT} = \frac{300}{0{,}189 * 333} = 4{,}767\ kg/m^3$

② specific weight $\Longrightarrow$ $\gamma_{CO_2} = \rho_{CO_2}\, g$

$= 4{,}767 * 9{,}81$

$= 46{,}764\ N/m^3$

Subject Date No.

* problem 2,6 :- determine the density and specific ~~heat~~ weight of methane gas at a pressure of 300 $kN/m^2$ absolute and 60°C ?

$R_{methane} = 0{,}518\ [kJ/kg \cdot k]$

$P = 300\,[kpa]$

$T = 60\,°C = 333\,°k$

$\rho_{He} = ??$ & $\gamma_{He} = ??$

sol :- $\rho = \frac{P}{RT} = \frac{300}{0{,}518 * 333} = 1{,}739\,[kg/m^3]$

$\gamma_{He} = \rho \cdot g = 1{,}739 * 9{,}81$
$= 17{,}06\ [N/m^3]$

Subject Date No.

**2.7** Natural gas is stored in a spherical tank at a temperature of 10°C. At a given initial time, the pressure in the tank is 100 kPa gage, and the atmospheric pressure is 100 kPa absolute. Some time later, after considerably more gas is pumped into the tank, the pressure in the tank is 200 kPa gage, and the temperature is still 10°C. What will be the ratio of the mass of natural gas in the tank when $p = 200$ kPa gage to that when the pressure was 100 kPa gage?

Situation: Natural gas (10°C) is stored in a spherical tank

Atmospheric pressure is $\boxed{100 \text{ kpa}}$

Initial tank pressure = 100 kpa

pumped into tank

Final tank pressure = 200 kpa

Find: Ratio of final mass to initial mass in the tank?

$\rho = \frac{m}{V}$ , $\rho = \frac{P}{RT}$ $\Rightarrow$ T constant $\Rightarrow$ isothermal

$m = \rho \forall$

$m = \frac{P}{RT} \forall$ $\Rightarrow$ Volume and gas templeture are constant

So $\Rightarrow$ $\frac{m_2}{m_1} = \frac{P_2}{P_1}$ $\Rightarrow$ $\frac{m_2}{m_1} = \frac{300}{200}$

$= \boxed{1{,}5}$

Subject Date No.

**2.8** At a temperature of 100°C and an absolute pressure of 5 atmospheres, what is the ratio of the density of water to the density of air, $\rho_w/\rho_a$?

$T = 100°C = 373°K$

$P = 5\ Atm = 5 * 101{,}32 = 506{,}600\ kpa$

$\frac{\rho_w}{\rho_{air}} = ??$ $R_{air} = 0{,}287 [kJ/kg°k]$

sol :- $\rho_{air} = \frac{P}{RT} = \frac{506{,}600}{0{,}287 * 373} = 4{,}73 [kg/m^3]$

$\rho_{water} = 958\ kg/m^3$

Ratio $\Rightarrow \frac{\rho_{H_2O}}{\rho_{air}} = \frac{958}{4{,}73} = 202$

**2.11** What are the specific weight and density of air at an absolute pressure of 600 kPa and a temperature of 50°C?

$P = 600 kpa$ , $T = 50°C = 323°K$ $\rho = ??$ $\gamma_{air} = ??$

$\rho_{air} = \frac{P}{RT} = \frac{600}{0{,}287 * 323} = 6{,}47\ kg/m^3$

$\gamma_{air} = \rho_{air} * g = 6{,}47 * 9{,}81$
$= 63{,}5\ N/m^3$

Subject Date No.

* Properties related to heat :-

$\boxed{C}$ ① specific heat [KJ/kg.k] الحرارة النوعية :- كمية الحرارة اللازمة لرفع اكغم
the amount of heat required to raise من المادة درجة مئوية واحدة .
the temprature of unit mass by one degree

$$\text{Heat energy} = m C \Delta T$$ * شرط أن يكون في نفس الطور *

$\boxed{U}$ ② Internal energy [J] ⟶ ⓐ kinematic energy K.E يتعلق بحركة الجزيئات
⟶ ⓑ potential energy P.E طاقة الترابط بين الجزيئات

$\boxed{H}$ ③ Enthalpy [J]

$$H = u + \frac{P}{\rho}$$

$v = \frac{1}{\rho}$ حجم نوعي

* تتغير بتغير درجة الحرارة بالنسبة للغازات

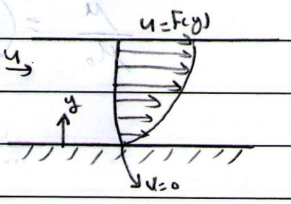
* تتغير بتغير الضغط ودرجة الحرارة بالنسبة للسوائل

*** Viscosity :- $\mu$ اللزوجة

$$\tau = \mu \frac{du}{dy}$$

shear stress

$\frac{du}{dy}$ ⇒ Rate of strain (تغير طول على طول)




$u = \frac{\Delta y}{\Delta t}$

$\tau \propto \frac{du}{dy}$

$$\mu = \frac{\tau}{du/dy}$$ ⇒ dynamic or absolute viscosity [pa.s]

| | |
|---|---|
| shear stress $\tau$ | $F // A$ |
| normal stress (pressure) بالنسبة للموائع | $F \perp A$ |
| strain تغير طول على طول | |

Poletechnic Lecture Note

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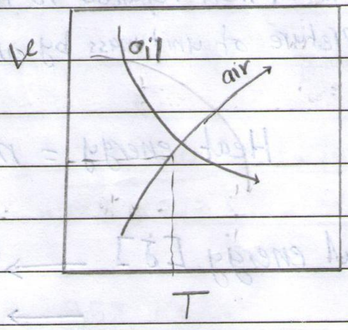
* Kinematic Viscosity :- $\boxed{\nu}$ ⇒ نيو

$$\nu = \frac{\mu}{\rho} \quad [m^2/s]$$

① The viscosity of liquids decreasing with increasing Tempreture

② The viscosity of gases increasing with increasing Tempreture




* to Find $\mu$ :-

① For liquid :-

$$\mu = C\, e^{b/T}$$

b و C ثابتين للصيغة الأولية

② For gases :- sutherland equation

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{\frac{3}{2}} * \left(\frac{T_0 + S}{T + S}\right)$$

يجب أن تكون اللزوجة معروفة
عند درجة حرارة محددة

* Table A2

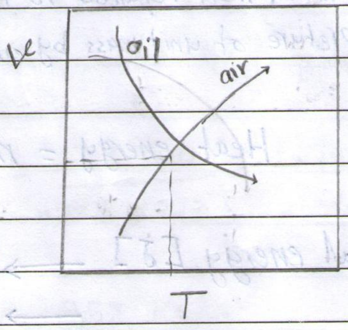
Subject Date No.

* Kinematic Viscosity :- $\boxed{\nu} \Rightarrow$ نيو

$$\nu = \frac{\mu}{\rho} \quad [m^2/s]$$

① The viscosity of liquids decreasing with increasing Tempreture

② The viscosity of gases increasing with increasing Tempreture




* to Find $\mu$ :-

① For liquid :-

$$\mu = c\, e^{b/T}$$

b و c ثابتين للصيغة الأولية

② For gases :- sutherland equation

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{\frac{3}{2}} * \left(\frac{T_0 + s}{T + s}\right)$$

يجب أن تكون اللزوجة معروفة
عند درجة حرارة محددة

* Table A2

Subject Date No.

* Elasticity:- المرونة ⇐ مقدار التغير في الحجم مقدار تغير حجم بـ pressure

تسمى بالفلود ⟸ Compressibity

$$\boxed{dp = -E_v \frac{d\forall}{\forall}} \quad [pa]$$

معامل المرونة

$E_v$ :- modulus of elasticity

$$\boxed{E_v = -\frac{dp}{d\forall/\forall}}$$ (كلما زاد الضغط تناقص الحجم)

$m = \rho\forall$

$dm = \rho d\forall + \forall d\rho$ $d\rho = 0$ ⇐ اذا m ثابت

$\rho d\forall = -\forall d\rho$

$\frac{d\rho}{\rho} = \frac{-d\forall}{\forall}$

$$\Rightarrow \boxed{dp = E_v \frac{d\rho}{\rho}}$$

① * For water $E_v = 2.2\,Gpa$

ex ⇒ $\Delta P = 1\,Mp$

$dp = E_v \frac{d\rho}{\rho}$

$1 \times 10^6 = 2.2 \times 10^9 \times \frac{d\rho}{\rho}$

$\frac{d\rho}{\rho} = 4.5 \times 10^{-4}$

$= 0.045\,\%$

incompressibel

⟹

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② * for gas :-

$$PV = mRT$$

$$P = \rho RT$$

* For isothermal $\Rightarrow$ T=constant

$$\frac{dP}{d\rho} = RT$$

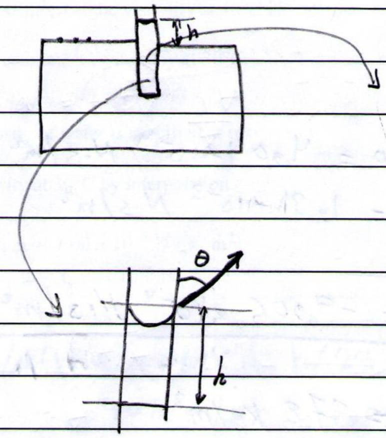
$$E_v = \frac{dP}{d\forall/\forall} = \frac{dP}{d\rho/\rho} = \rho\frac{dP}{d\rho}$$

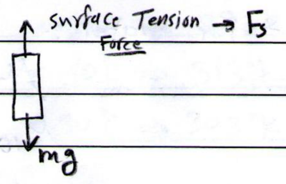
$$E_v = \rho RT = P$$

$$\therefore \boxed{E_v = P}$$

Subject Date No.

* surface tension :- $\boxed{\sigma}$ التوتر السطحي $[N/m]$




$F_s = \sigma \, L_w \Rightarrow$ الطول المبلل

(الطول ما بين المماس للمائع والسطح)

$$\cos\Theta * F_s = w = \sigma \underbrace{[2\pi r]}_{\cos\Theta * \text{محيط الدائرة}} = mg$$

$$= \rho \forall g = \rho A h g$$

$$h = \frac{2\cancel{\pi}\cancel{r}\sigma \cos\Theta}{\rho[\cancel{\pi} r^{\cancel{2}}] g}$$

$$= \frac{2\sigma \cos\Theta}{r\rho g} = \frac{2\sigma}{r\gamma} \cos\Theta$$

Subject Problem 2,18 Date No.

**2.18** What is the change in the viscosity and density of water between 10°C and 70°C? What is the change in the viscosity and density of air between 10°C and 70°C? Assume standard atmospheric pressure ($p = 101\ kN/m^2$ absolute).

Sol:-

* For water:-

$$\mu_{70} = 4,04 * 10^{-4}\ N.s/m^2$$

$$\mu_{10} = 1,31 * 10^{-3}\ N.s/m^2$$

$$\boxed{\Delta\mu = -9,06 * 10^{-4}\ N.s/m^2}$$

$$\rho_{70} = 978\ kg/m^3$$

$$\rho_{10} = 1000\ kg/m^3$$

$$\boxed{\Delta\rho = -22\ kg/m^3}$$

* For Air:-

$$\mu_{70} = 2,04 * 10^{-5}\ N.s/m^2$$

$$\mu_{10} = 1,76 * 10^{-5}\ N.s/m^2$$

$$\boxed{\Delta\mu = 2,8 * 10^{-6}\ N.s/m^2}$$

$$\rho_{70} = 1,03\ kg/m^3$$

$$\rho_{10} = 1,25\ kg/m^3$$

$$\boxed{\Delta\rho = -0,22\ kg/m^3}$$

Subject ex: 2,2 Date No.

EXAMPLE 2.2 CALCULATING VISCOSITY OF LIQUID AS A FUNCTION OF TEMPERATURE

The dynamic viscosity of water at 20°C is $1.00 \times 10^{-3}$ N · s/m², and the viscosity at 40°C is $6.53 \times 10^{-4}$ N · s/m².
Using Eq. (2.9), estimate the viscosity at 30°C.

*Problem Definition*

**Situation:** Viscosity of water is specified at two temperatures.

**Find:** The viscosity at 30°C by interpolation.

**Properties:**

a) Water at 20°C, $\mu = 1.00 \times 10^{-3}$ N · s/m².

b) Water at 40°C, $\mu = 6.53 \times 10^{-4}$ N · s/m².

* For water

$$\mu = C e^{b/T}$$

* 20°C ⇒ 293°K
* 40°C ⇒ 313°K
* 30°C ⇒ 303°K

* $\ln \mu = \ln C + (b/T)$

ⓐ $\ln(1 \times 10^{-3}) = \ln C + b/293$

$-6,908 = \ln C + 0,00341\, b \Rightarrow ①$

ⓑ $\ln(6,53 \times 10^{-4}) = \ln C + b/313$

$-7,334 = \ln C + 0,00319\, b \Rightarrow ②$

⇒ solution $\underline{\ln C}$ and $\underline{b}$ :- $\ln C = -13,51$ , $b = 1936$ K #

$C = e^{-13,51} = 1,357 \times 10^{-6}$ #

$$\boxed{\mu = (1,357 \times 10^{-6}) \times e^{1936/T}}$$

$\mu_{30°C} = 1,357 \times 10^{-6} \times e^{1936/303}$

$$\boxed{\mu = 8,083 \times 10^{-4}\ N \cdot s/m^2}$$

Subject ex 2.3 Date No.

**EXAMPLE 2.3 MODELING A BOARD SLIDING ON A LIQUID LAYER**

A board 1 m by 1 m that weighs 25 N slides down an inclined ramp (slope = 20°) with a velocity of 2.0 cm/s. The board is separated from the ramp by a thin film of oil with a viscosity of 0.05 $N \cdot s/m^2$. Neglecting edge effects, calculate the space between the board and the ramp.

*Problem Definition*

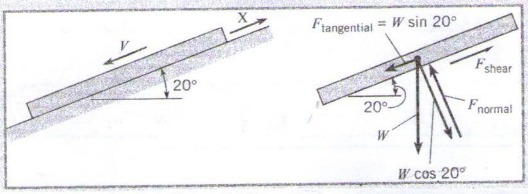
**Situation:** A board is sliding down a ramp, on a thin film of oil.

**Find:** Space (in m) between the board and the ramp.

**Assumptions:** A linear velocity distribution in the oil.

**Properties:** Oil, $\mu = 0.05$ $N \cdot s/m^2$.

**Sketch:** $V = 0.02$ m/s, $W = 25$ N




$F_{tangential} = F_{shear}$

$W \sin 20 = \tau A$

$W \sin 20 = \mu \frac{dv}{dy} A$

* substition of dv/dy as ΔV/Δy

$W \sin 20 = \mu \frac{\Delta V}{\Delta y} A$

$\Delta y = \frac{\mu \Delta V A}{W \sin 20}$

$\Delta y = \frac{0.05 * 0.02 * 1}{25 \sin 20}$

$\Delta y = 0.000117$ m

$\boxed{\Delta y = 0.117 \text{ mm}}$ #

N O T E B O O K

Subject Date No.

**EXAMPLE 2.4 CAPILLARY RISE IN A TUBE**

To what height above the reservoir level will water (at 20°C) rise in a glass tube, such as that shown in Fig. 2.7, if the inside diameter of the tube is 1.6 mm?

*Problem Definition*

**Situation:** A glass tube of small diameter placed in an open reservoir of water induces capillary rise.

**Find:** The height the water will rise above the reservoir level.

**Sketch:** See Figure 2.7.

**Properties:** Water (20 °C), Table A.5, $\sigma = 0.073$ N/$m$; $\gamma = 9790$ N/m$^3$.

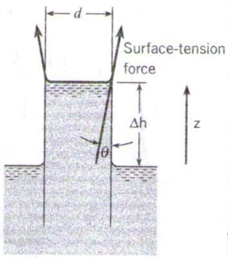
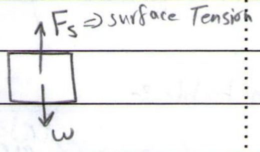



**Figure 2.7**

solution:-




$$F_s - W = 0$$

$$\sigma[2\pi r]\cos\theta - mg = 0$$

$$2\sigma\pi r\cos\theta - \rho \forall g = 0$$

$$2\sigma\pi r\cos\theta - \gamma \forall = 0$$

$$\cos\theta \approx 1$$

$$2\sigma\pi r - \gamma A \Delta H = 0$$

$$2\sigma\pi r - \gamma(\Delta H)\left(\frac{\pi d^2}{4}\right) = 0$$

$$\boxed{\Delta h = \frac{4\sigma}{\gamma d} = 18.26\ mm}$$ #

oR: $$\boxed{\Delta h = \frac{2\sigma}{\gamma r}\cos\theta = 18.26\ mm}$$

Subject Date No.

* problem 2.19 :- determine the change in the Kinematic Viscosity of air that is heated from 10°C to 60°C. Assume standard atmospheric pressure?

$\nu_{60} = 1.89 \times 10^{-5}$ [m²/s] , $\nu_{10} = 1.41 \times 10^{-5}$ [m²/s]

$\Delta\nu_{Air\ 10 \to 60} = (1.89 - 1.41) \times 10^{-6}$

$= 4.8 \times 10^{-6}$ m²/s

2.21 What is the ratio of the dynamic viscosity of air to that of water at standard pressure and a temperature of 20°C? What is the ratio of the kinematic viscosity of air to that of water for the same conditions?

From table :-

$\mu_{air\ 20°C} = 1.81 \times 10^{-5}$ N.s/m² , $\nu = 1.51 \times 10^{-5}$ m²/s

$\mu_{water\ 20°C} = 1 \times 10^{-3}$ N.s/m² , $\nu = 1 \times 10^{-6}$ m²/s

$\frac{\mu_{air}}{\mu_{water}} = 1.81 \times 10^{-2}$ [N.s/m²]

$\frac{\nu_{air}}{\nu_{water}} = 15.1$ [m²/s]

NOTEBOOK

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**2.22** Using Sutherland's equation and the ideal gas law, develop an expression for the kinematic viscosity ratio $\nu/\nu_0$ in terms of pressures $p$ and $p_0$ and temperatures $T$ and $T_0$, where the subscript 0 refers to a reference condition.

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2}\left(\frac{T_0+S}{T+S}\right)$$

$$\frac{\mu}{\mu_0} = \frac{\nu}{\nu_0} * \frac{\rho}{\rho_0} \Rightarrow \boxed{\rho = \frac{P}{RT}}$$

$$\boxed{\frac{\mu}{\mu_0} = \frac{\nu}{\nu_0} * \frac{P}{P_0} * \frac{T_0}{T}}$$

$$\Rightarrow \frac{\nu}{\nu_0} * \frac{P}{P_0} * \frac{T_0}{T} = \left(\frac{T}{T_0}\right)^{3/2}\left(\frac{T_0+S}{T+S}\right)$$

$$\boxed{\frac{\nu}{\nu_0} = \frac{P_0}{P} * \left(\frac{T}{T_0}\right)^{5/2}\left(\frac{T_0+S}{T+S}\right)}$$

kinematic viscosity ratio $\nu/\nu_0$

**2.23** The dynamic viscosity of air at 15°C is $1.78 \times 10^{-5}$ $N \cdot s/m^2$. Using Sutherland's equation, find the viscosity at 100°C.

$S_{air} = 110{,}4\ K$

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2}\left(\frac{T_0+S}{T+S}\right) = \left(\frac{373}{288}\right)^{3/2}\left(\frac{288+110{,}4}{373+110{,}4}\right)$$

$$= 1{,}214$$

$$\mu = 1{,}214\,\mu_0 = 1{,}214 * (1{,}78 * 10^{-5})$$

$$= 2{,}162 * 10^{-5} [N \cdot s/m^2]$$

Subject Date No.

**2.24** The kinematic viscosity of methane at 15°C and atmospheric pressure is $1.59 \times 10^{-5}$ $m^2/s$. Using Sutherland's equation and the ideal gas law, find the kinematic viscosity at 200°C and 2 atmospheres.

$S = 198\,K$

$$\frac{\nu}{\nu_0} = \frac{P_0}{P}\left(\frac{T}{T_0}\right)^{5/2}\left(\frac{T_0+S}{T+S}\right)$$

$\nu = \frac{\mu}{\rho}$

$\mu =$

$$= \frac{1}{2}\left(\frac{473}{288}\right)^{5/2}\left(\frac{288+198}{473+198}\right)$$

$$= 1{,}252$$

$$\nu = 1{,}252 * \nu_0$$

$$= 1{,}252 * 1{,}59 \times 10^{-5}$$

$$= 1{,}99 * 10^{-5}\,[m^2/s]$$

**2.25** The dynamic viscosity of nitrogen at 59°F is $3.59 \times 10^{-7}$ $lbf \cdot s/ft^2$. Using Sutherland's equation, find the dynamic viscosity at 200°F.

$S = 192$

Rankine:

$°R = °F + 460$

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2}\left(\frac{T_0+S}{T+S}\right)$$

$$= \left(\frac{660}{519}\right)^{3/2} * \left(\frac{519+192}{660+192}\right)$$

$$= 1{,}197$$

$$\mu = 1{,}197 * (3{,}59 * 10^{-7})$$

$$= 4{,}3 * 10^{-7}\,[lbf \cdot s/ft^2]$$

Subject Date No.

**2.26** The kinematic viscosity of helium at 59°F and 1 atmosphere is $1.22 \times 10^{-3}$ $ft^2/s$. Using Sutherland's equation and the ideal gas law, find the kinematic viscosity at 30°F and a pressure of 1.5 atmospheres.

$S = 143°R$

$$\frac{\nu}{\nu_0} = \frac{P_0}{P}\left(\frac{T}{T_0}\right)^{5/2}\left(\frac{T_0+S}{T+S}\right)$$

$$= \frac{1,5}{1}\left(\frac{490}{519}\right)^{5/2}\left(\frac{519+143}{490+143}\right) = 1,359$$

$$\nu = 1,359 * \left(1,22 \times 10^{-3}\ [ft^2/s]\right)$$

$$= 1,658 \times 10^{-3}\ [ft^2/s]$$

**2.27** The absolute viscosity of propane at 100°C is $1.00 \times 10^{-5}$ $N \cdot s/m^2$ and at 400°C is $1.72 \times 10^{-5} N \cdot s/m^2$. Find Sutherland's constant for propane.

$100°C \Rightarrow 373K$

$400°C \Rightarrow 673K$

$$\frac{T}{T_0} = \frac{673}{373} = 1,8042$$

$$\frac{\mu}{\mu_0} = \frac{1,72 \times 10^{-5}}{1 \times 10^{-5}} = 1,72$$

$$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2}\left(\frac{T_0+S}{T+S}\right)$$

$$\frac{1,72}{2,423} = \frac{T_0+S}{T+S}$$

$$0,7097 = \frac{373+S}{673+S}$$

$$0,7097(673+S) = 373+S$$

$$477,657 + 0,7097S = 373+S$$

$$104,657 = 0,29\ S$$

$$\boxed{S = 360,5\ K}$$

Subject Date No.

**2.28** Ammonia is very volatile, so it may be either a gas or a liquid at room temperature. When it is a gas, its absolute viscosity at 68°F is $2.07 \times 10^{-7}$ lbf · s/ft² and at 392°F is $3.46 \times 10^{-7}$ lbf · s/ft². Using these two data points, find Sutherland's constant for ammonia.

$T \Rightarrow 392 F \Rightarrow 852 R$

$T_0 \Rightarrow 68 F \Rightarrow 528 R$

$\frac{T}{T_0} = 1{,}6136$

$\frac{\mu}{\mu_0} = \frac{3{,}46 \times 10^{-7}}{2{,}07 \times 10^{-7}} = 1{,}671$

$\frac{\mu}{\mu_0} = \left(\frac{T}{T_0}\right)^{3/2} \left(\frac{T_0 + S}{T + S}\right)$

$1{,}671 = 2{,}05 \left(\frac{528 + S}{852 + S}\right)$

$0{,}815 = \frac{528 + S}{852 + S}$

$0{,}815 (852 + S) = 528 + S$

$694{,}38 + 0{,}815 S = 528 + S$

$166{,}38 = 0{,}185 S$

$\boxed{S \approx 902 \,°R}$

Subject Date No.

**2.31** Two plates are separated by a 1/8-in. space. The lower plate is stationary; the upper plate moves at a velocity of 25 ft/s. Oil (SAE 10W-30, 150°F), which fills the space between the plates, has the same velocity as the plates at the surface of contact. The variation in velocity of the oil is linear. What is the shear stress in the oil?

$0,125\,in \Rightarrow 0,125/12 = 0,01\,ft$

$\tau\ ??$

$\mu_{oil} = 5,2\times10^{-4}\ lbf\cdot s/ft^2$

Rate of strain $\Rightarrow \dfrac{du}{dy} = \dfrac{\Delta u}{\Delta y} = \dfrac{25\,ft/s}{(0,01)\,ft}$

$= 2400\ s^{-1}$

$\tau = \mu\left(\dfrac{du}{dy}\right)$

$= \left(5,2\times10^{-4}\ \dfrac{lbf\cdot s}{ft^2}\right)\times\left(2400\ \dfrac{1}{s}\right) = 1,248\ \dfrac{lbf}{ft^2}$

$$\boxed{\tau \approx 1,25\,[lbf/ft^2]}$$

Subject Date No.

**2.32** Find the kinematic and dynamic viscosities of air and water at a temperature of 40°C (104°F) and an absolute pressure of 170 kPa (25 psia).

$\mu_{Air} = 1{,}91 * 10^{-5} [N.s/m^2]$

$\mu_{H_2O} = 6{,}53 * 10^{-4} [N.s/m^2]$

$\rho_{H_2O} = 992 [kg/m^3]$

Ⓐ For Air:-

$$\rho_{air} = \frac{P}{RT} = \frac{170}{0{,}287 * 313{,}2} = 1{,}89\ kg/m^3$$

$$\mu_{air} = 1{,}91 * 10^{-5}\ N.s/m^2$$

$$\nu = \frac{\mu}{\rho} = \frac{1{,}91 * 10^{-5}}{1{,}89} = 10{,}1 * 10^{-6}\ m^2/s$$

Ⓑ for water

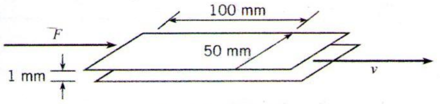
$$\mu_{water} = 6{,}53 * 10^{-5}\ N.s/m^2$$

$$\nu = \frac{\mu}{\rho} = \frac{6{,}53 * 10^{-5}}{992} = 6{,}58 * 10^{-7}\ [m^2/s]$$

N O T E B O O K

Subject Date No.

**2.33** The sliding plate viscometer shown below is used to measure the viscosity of a fluid. The top plate is moving to the right with a constant velocity of 10 m/s in response to a force of 3 N. The bottom plate is stationary. What is the viscosity of the fluid? Assume a linear velocity distribution.




PROBLEM 2.33

$u = 10\ m/s$

$F = 3N$

$$\tau = \mu \frac{du}{dy} \qquad \tau = \frac{F}{A} = \frac{3}{0{,}1 * 0{,}05} = 600$$

$$\mu = \frac{\tau}{du/dy} = \frac{600}{(10/1*10^{-3})} = 0{,}06\,[N.s/m^2]$$

Subject Date No.

**2.34** The velocity distribution for water (20°C) near a wall is given by $u = a(y/b)^{1/6}$, where $a = 10$ m/s, $b = 2$ mm, and $y$ is the distance from the wall in mm. Determine the shear stress in the water at $y = 1$ mm.

$$u = a\left(\frac{y}{b}\right)^{1/6} \qquad T = 20^\circ C \quad a = 10\ m/s \quad b = 2\ mm$$

$$\frac{du}{dy} = \frac{1}{6}\frac{a}{b}\left(\frac{y}{b}\right)^{-5/6} \qquad \tau\big|_{y=1mm} = ?? \qquad \tau = \mu \frac{du}{dy}$$

$$= \frac{a}{6b}\left(\frac{b}{y}\right)^{5/6}$$

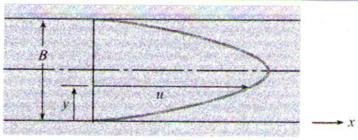
$$= \frac{10}{6 \times 0{,}002} \times \left(\frac{2mm}{1mm}\right)^{5/6} = 1485\ s^{-1}$$

$$\tau\big|_{y=1mm} = \mu \frac{du}{dy}$$

$$= (1 \times 10^{-3}\ Ns/m^2)(1485\ s^{-1})$$

$$= 1{,}485\ Pa$$

**2.35** The velocity distribution for the flow of crude oil at 100°F ($\mu = 8 \times 10^{-5}$ lbf · s/ft$^2$) between two walls is shown, and is given by $u = 100y(0.1 - y)$ ft/s, where $y$ is measured in feet and the space between the walls is 0.1 ft. Plot the velocity distribution and determine the shear stress at the walls.




PROBLEMS 2.35, 2.36, 2.37

$$u = 100\,y\,(0.1 - y) = 10y - 100y^2$$

$$\boxed{\frac{du}{dy} = 10 - 200y}$$

$$\left.\frac{du}{dy}\right|_{y=0} = 10\ s^{-2}$$

$$\left.\frac{du}{dy}\right|_{y=0.1} = -10\ s^{-1}$$

$$\tau_0 = \mu \frac{du}{dy} = (8\times10^{-5})(10)$$
$$= 8\times10^{-4}\ \text{lbf/ft}^2$$

$$\tau_{0.1} = 8\times10^{-4}\ \text{lbf/ft}^2$$

Subject Date No.

**2.36** A liquid flows between parallel boundaries as shown above. The velocity distribution near the lower wall is given in the following table:

| $y$ in mm | $V$ in m/s |
|---|---|
| 0.0 | 0.00 |
| 1.0 | 1.00 |
| 2.0 | 1.99 |
| 3.0 | 2.98 |

a. If the viscosity of the liquid is $10^{-3}$ N · s/m², what is the maximum shear stress in the liquid?
b. Where will the minimum shear stress occur?

(a) $\mu = 10^{-3}$ N.s/m²

$\tau_{max} = \mu \dfrac{du}{dy}$ ≈ next to wall

$\tau_{max} = 10^{-3} \left( \dfrac{1 \text{ m/s}}{0.001} \right) = 1 \text{ N/m}^2$

(b) minimum $\tau$ will be zero ⇒ velocity gradiant is zero

Subject Date No.

**2.37** Suppose that glycerin is flowing ($T = 20°C$) and that the pressure gradient $dp/dx$ is $-1.6\ kN/m^3$. What are the velocity and shear stress at a distance of 12 mm from the wall if the space $B$ between the walls is 5.0 cm? What are the shear stress and velocity at the wall? The velocity distribution for viscous flow between stationary plates is

$$u = -\frac{1}{2\mu}\frac{dp}{dx}(By - y^2)$$

$20°C \Rightarrow \mu = 1{,}41\ N.s/m^2$

(a) $u$ and $\tau \Big|_{y=12\,mm}$ ?

Velocity:

$$u = \frac{-1}{2\mu}\frac{dp}{dx}(By - y^2)$$

$$= -\frac{1}{2(1{,}41)}(-1600)\left((0{,}05)(0{,}012) - (0{,}012)^2\right)$$

$$= 0{,}2587\ m/s$$

$$\approx 0{,}259\ m/s$$

Rate of strain:

$$\frac{du}{dy} = \frac{d}{dy}\left(\frac{1}{-2\mu}\frac{dp}{dx}(By - y^2)\right)$$

$$= \left(\frac{-1}{2\mu}\right)\left(\frac{dp}{dx}\right)\frac{d}{dy}(By - y^2)$$

$$= \left(\frac{-1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y)$$

$$\frac{du}{dy}\Big|_{y=12\,mm} = \left(\frac{-1}{2(1{,}41)}\right)(-1600)(0{,}05 - 2 * 0{,}012)$$

$$= 14{,}75\ s^{-1}$$

Shear stress $\tau = \mu \frac{du}{dy}$

$$= (1{,}41)(14{,}75) = 20{,}798\ Pa$$

$$\approx 20{,}8\ Pa$$

Subject Date No.

(b) Velocity $|_{y=0mm}$

$$u = \frac{-1}{2\mu}\frac{dp}{dx}(By - y^2)$$

$$= \frac{-1}{2(1,41)}(-1600)((0,05(0) - (0)^2)$$

$$= 0\ m/s$$

Rate of strain $|_{y=0mm}$

$$\frac{du}{dy} = \left(\frac{-1}{2\mu}\right)\left(\frac{dp}{dx}\right)(B - 2y)$$

$$= \left(\frac{-1}{2(1,41)}\right)(-1600)(0,05 - 2\times 0)$$

$$= 28,37\ s^{-1}$$

Shear stress $|_{y=0mm}$

$$\tau = \mu\frac{du}{dy}$$

$$= (1,41)(28,37)$$

$$= 40pa$$

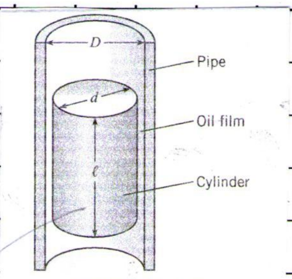
**2.39** Consider the ratio $\mu_{100}/\mu_{50}$, where $\mu$ is the viscosity of oxygen and the subscripts 100 and 50 are the temperatures of the oxygen in degrees Fahrenheit. Does this ratio have a value (a) less than 1, (b) equal to 1, or (c) greater than 1?

Correct choice is $\boxed{C}$

because the viscosity of gases increases with temperature $\frac{\mu_{100}}{\mu_{50}} > 1$

Subject Date No.

**2.40** This problem involves a cylinder falling inside a pipe that is filled with oil, as depicted in the figure. The small space between the cylinder and the pipe is lubricated with an oil film that has viscosity $\mu$. Derive a formula for the steady rate of descent of a cylinder with weight $W$, diameter $d$, and length $\ell$ sliding inside a vertical smooth pipe that has inside diameter $D$. Assume that the cylinder is concentric with the pipe as it falls. Use the general formula to find the rate of descent of a cylinder 100 mm in diameter that slides inside a 100.5 mm pipe. The cylinder is 200 mm long and weighs 15 N. The lubricant is SAE 20W oil at 10°C.



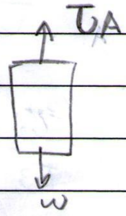

PROBLEM 2.40

$w = 15\ N$

$\mu_{/10°C} = 0.3\ pa.s$

$D = 100.5\ mm$

$d = 100\ mm \qquad L = 200\ mm$




$\tau A = w$

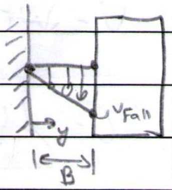
$\mu \dfrac{du}{dy} [\pi d L] = w$

$\mu \left[\dfrac{2 V_{Fall}}{D-d}\right] * [\pi d L] = w$

$0.3 \left[\dfrac{2 * V_{Fal}}{0.5 * 10^{-3}}\right] * [\pi * 0.1 * 0.2] = 15$

$24 \pi V_{Fall} = 15$

$V_{Fall} = 0.198\ m/s$




$u = \dfrac{y}{B} V_F = \dfrac{y V_F}{(D-d)/2}$

$u = \left[\dfrac{2 V_F}{D-d}\right] y$

$\dfrac{du}{dy} = \dfrac{2 V_F}{D-d}$

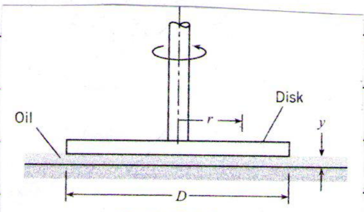
Subject Date No.

**2.41** The device shown consists of a disk that is rotated by a shaft. The disk is positioned very close to a solid boundary. Between the disk and the boundary is viscous oil.

a. If the disk is rotated at a rate of 1 rad/s, what will be the ratio of the shear stress in the oil at $r = 2$ cm to the shear stress at $r = 3$ cm? $\tau_2/\tau_3$

b. If the rate of rotation is 2 rad/s, what is the speed of the oil in contact with the disk at $r = 3$ cm?

c. If the oil viscosity is 0.01 $N \cdot s/m^2$ and the spacing $y$ is 2 mm, what is the shear stress for the conditions noted in part (b)?




PROBLEM 2.41

$$\tau = \mu \frac{du}{dy} \qquad \frac{du}{dy} = \frac{V}{y} = \frac{\omega r}{y}$$

linear velocity

ⓐ $$\frac{\tau_2}{\tau_3} = \frac{(\cancel{\mu} * 1 * \frac{2}{\cancel{y}})}{(\cancel{\mu} * 1 * \frac{3}{\cancel{y}})} = \frac{2}{3} = 0{,}667$$

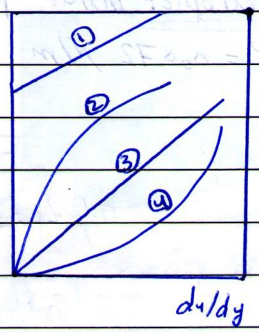
ⓑ $$V = \omega r = 2 * 0{,}03 = 0{,}06 \; m/s$$

ⓒ $$\tau = \mu \frac{du}{dy} = 0{,}01 * \frac{0{,}06}{0{,}002} = 0{,}3 \; N/m^2$$

Subject: Newtonian fluid and Non newtonian fluid    Date    No.

Newtonian fluid

$\Rightarrow \tau = \mu \dfrac{du}{dy}$ تنطبق عليه




1. $\tau = S \dfrac{du}{dy} + C \Rightarrow$ non newtonian

2 + 4 non newtonian fluid $\Rightarrow$ non linear

3 newtonian $\Rightarrow$ linear $\Rightarrow \tau = \mu \dfrac{du}{dy}$

Subject Date No.

ex:- water droplet inside present 50 kpa find the diameter?

$\sigma = 0.072$ N/m

$$F_s = P.A$$

$$\sigma \, L_w \, \cancel{\cos\theta} = P(\pi r^2)$$

$$\sigma(2\cancel{\pi r}) = P \cancel{\pi} r^{\cancel{2}}$$

$$r = \frac{2\sigma}{P} = \frac{2 \times 0.072}{50} = 0.00288 \text{ m} = 2.88 \text{ mm}$$

$$D = 5.76 \text{ mm}$$

$$\boxed{r = \frac{2\sigma}{P}} \rightarrow \text{droplet}$$

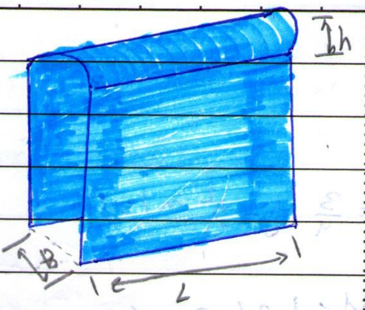
$$\boxed{P = \frac{2\sigma}{r}}$$

* for bubble $\Rightarrow$ $\sigma \underset{(4\pi r)}{L_w} = P.A$

$$\boxed{P = \frac{4\sigma}{r}} \rightarrow \text{bubble}$$

Subject　　　　　　　　　　　　　　Date　　　　　　No.

ex:




$F_s = W$

$\sigma\, L w \cos\theta = mg = \rho V g$　　　$B = 1\,mm$

$\sigma * [2\cancel{L}] = \rho\,[\cancel{L} B h]\, g$

(لأن الماء مع 2L)

$= \gamma B h$

$h = \dfrac{2\sigma}{\gamma B}$

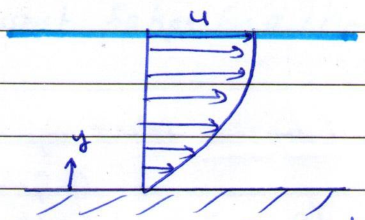
$h = \dfrac{2 * 0.073}{9810 * 1 * 10^{-3}} = 14.9 \times 10^{-3}\, m$

$= 14.9\ mm$

N O T E B O O K

Subject Date No.

ex:-




flow over flat plate

$u = \frac{3}{4} y - y^2 \ m/s$

$\mu = 1,36 \ pa.s$

Find $\tau$ at $y = 15 \ cm$?

$$\tau\big|_{0,15} = \mu \frac{du}{dy}\bigg|_{y=0,15}$$

$$= \mu \left[\frac{3}{4} - 2y\right]\bigg|_{y=0,15}$$

$$= 1,36 \left[0,75 - 0,3\right]$$

$$= 0,613 \ Pa$$

Subject Date No.

**2.45** A pressure of $2 \times 10^6\ \mathrm{N/m^2}$ is applied to a mass of water that initially filled a 2000 $\mathrm{cm^3}$ volume. Estimate its volume after the pressure is applied.

From Table A5 $E_{H_2O} = 2,2 \times 10^9$ pa

$$dp = -E_v \frac{d\forall}{\forall}$$

$$2 \times 10^6 = -2,2 \times 10^9 \frac{\Delta\forall}{2000\ \mathrm{cm^3}}$$

$$\Delta\forall = -1,818\ \mathrm{cm^3}$$

$$\forall_{Final} = \forall + \Delta\forall$$

$$= (2000 - 1,81)\ \mathrm{cm^3}$$

$$= 1998,19\ \mathrm{cm^3}$$

**2.46** Calculate the pressure increase that must be applied to water to reduce its volume by 2%.

$$dp = -E_v \frac{d\forall}{\forall}$$

$$= -2,2 \times 10^9 * \left(\frac{-0,02 * \forall}{\forall}\right)$$

$$= -2,2 \times 10^9 \times -0,02$$

$$= 4,4 \times 10^7\ \mathrm{pa}$$

$$= 44\ \mathrm{MPa}$$

Subject Date No.

**2.49** Which of the following is the formula for the gage pressure within a very small spherical droplet of water:
(a) $p = \sigma/d$, (b) $p = 4\sigma/d$, or (c) $p = 8\sigma/d$?

spherical droplet

$$F_s = P \cdot A$$

$$\sigma[2\cancel{\pi} \cancel{r}] = P(\cancel{\pi} r^{\cancel{2}})$$

$$P = \frac{2\sigma}{r_{=\frac{d}{2}}} = \boxed{\frac{4\sigma}{d}}$$

**2.50** A spherical soap bubble has an inside radius $R$, a film thickness $t$, and a surface tension $\sigma$. Derive a formula for the pressure within the bubble relative to the outside atmospheric pressure. What is the pressure difference for a bubble with a 4 mm radius? Assume $\sigma$ is the same as for pure water.

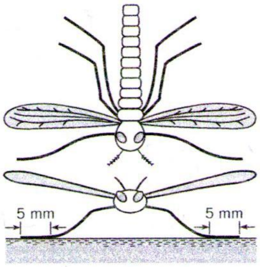
$\sigma = 7{,}3 \times 10^{-2}$ N/m

$$P = \frac{4\sigma}{r} = \frac{4 * 7{,}3 * 10^{-2}}{0{,}004}$$

$$= 73 \text{ N/m}^2$$

Subject Date No.

**2.51** A water bug is suspended on the surface of a pond by surface tension (water does not wet the legs). The bug has six legs, and each leg is in contact with the water over a length of 5 mm. What is the maximum mass (in grams) of the bug if it is to avoid sinking?




PROBLEM 2.51

$\sigma = 0.073\ N/m$

$F_s = \omega$

$\sigma\,[2\,leg * 6\,legs * L] = m\,g$

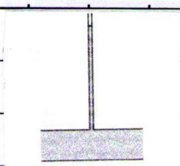
$\sigma\,[12 * 0.005] = m * 9.81$

$m = \dfrac{0.073 * 12 * 0.005}{9.81}$

$m = 0.447 \times 10^{-3}\ kg$

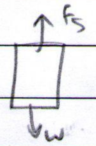
N O T E B O O K

**2.52** A water column in a glass tube is used to measure the pressure in a pipe. The tube is 1/4 in. (6.35 mm) in diameter. How much of the water column is due to surface-tension effects? What would be the surface-tension effects if the tube were 1/8 in. (3.2 mm) or 1/32 in. (0.8 mm) in diameter?



PROBLEM 2.52

$$F_s = W$$




$$\sigma\, l_w \cos\theta = w$$

$$\sigma[2\pi r] = mg = \rho \forall g$$

$$= \gamma \forall$$

$$\sigma[2\pi r] = \gamma A \Delta h = \gamma \left[\frac{\pi d^2}{4}\right] \Delta h$$

$$\boxed{\Delta h = \frac{4\sigma}{\gamma d}}$$

$$\Delta h = \frac{4\sigma}{\gamma d} = \frac{4 \times 0{,}005}{62{,}4 \times d} = \frac{3{,}21 \times 10^{-4}}{d}\ \text{ft}$$

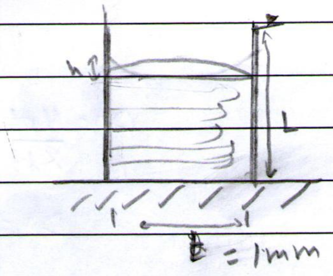
$$d = 1/4\ \text{in} = 1/48\ \text{ft} \Rightarrow \Delta h = \frac{3{,}21 \times 10^{-4}}{1/48} = 0{,}0154\ \text{ft} = 0{,}185\ \text{in}$$

$$d = 1/8\ \text{in} = 1/96\ \text{ft} \Rightarrow \Delta h = \frac{3{,}21 \times 10^{-4}}{1/96} = 0{,}0308\ \text{ft} = 0{,}369\ \text{in}$$

$$d = 1/32\ \text{in} = 1/384\ \text{ft} \Rightarrow \Delta h = \frac{3{,}21 \times 10^{-4}}{1/384} = 0{,}123\ \text{ft} = 1{,}48\ \text{in}$$

Subject Date No.

2.53 Calculate the maximum capillary rise of water between two vertical glass plates spaced 1 mm apart.




$$F_s = W$$

$$\sigma (Lw) = W$$

$$\sigma (2L) = \forall \gamma$$

$$\sigma (2L) = (hLt)\gamma$$

$$h = \frac{2\sigma}{\gamma t} = \frac{2 \times (7.3 \times 10^{-2})}{9810 \times 0.001}$$

$$= 0.0149 \text{ m}$$

$$= 14.9 \text{ mm}$$

2.54 What is the pressure within a 1 mm spherical droplet of water relative to the atmospheric pressure outside?

$$F_s = P \cdot A$$

$$\sigma [2\pi r] = p[\pi r^2]$$

$$P = \frac{2\sigma}{r}$$

$$P = \frac{2 \times 7.3 \times 10^{-2}}{0.5 \times 10^{-3}} = 292 \text{ N/m}^2$$

Subject Date No.

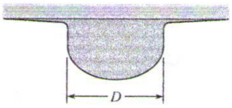
**2.55** By measuring the capillary rise in a tube, one can calculate the surface tension. The surface tension of water varies linearly with temperature from 0.0756 N/m at 0°C to 0.0589 N/m at 100°C. Size a tube (specify diameter and length) that uses capillary rise of water to measure temperature in the range from 0°C to 100°C. Is this design for a thermometer a good idea?

$$\Delta h = \frac{4\sigma}{\gamma d}$$

$$\Delta h = \frac{4 \times 0.0167}{9810 \times d} = \frac{6.8 \times 10^{-6}}{d}$$

The change column elevation for 1mm dia tube would 6.8 mm

**2.57** A drop of water at 20°C is forming under a solid surface. The configuration just before separating and falling as a drop is shown in the figure. Assume the forming drop has the volume of a hemisphere. What is the diameter of the hemisphere just before separating?




PROBLEM 2.57

Table A5:

20°C $\Rightarrow \gamma = 9790\ N/m^3$

$\sigma = 0.073\ N/m$

$$F_s = W$$

$$\sigma(\pi D) = \gamma \forall$$

$$\sigma(\pi D) = \gamma\left(\frac{\pi d^3}{12}\right) \quad \frac{1}{2} \times \frac{1}{6}\pi D^3$$

$$d^2 = \frac{12\sigma}{\gamma}$$

$$= \frac{12 \times 0.073}{9790} = 8.948 \times 10^{-5}\ m^2$$

$$d = 9.459 \times 10^{-3}\ m$$

$$D = 9.46\ mm$$

Poletechnic Lecture Note

Subject Date No.

## EXAMPLE 1.5 Newtonian Fluid Shear Stress

**GIVEN** The velocity distribution for the flow of a Newtonian fluid between two wide, parallel plates (see Fig. E1.5*a*) is given by the equation

$$u = \frac{3V}{2}\left[1 - \left(\frac{y}{h}\right)^2\right]$$

where $V$ is the mean velocity. The fluid has a viscosity of $0.04\ \text{lb} \cdot \text{s/ft}^2$. Also, $V = 2\ \text{ft/s}$ and $h = 0.2$ in.

**FIND** Determine: (a) the shearing stress acting on the bottom wall, and (b) the shearing stress acting on a plane parallel to the walls and passing through the centerline (midplane).

### SOLUTION

For this type of parallel flow the shearing stress is obtained from Eq. 1.9,

$$\tau = \mu \frac{du}{dy} \qquad (1)$$

Thus, if the velocity distribution $u = u(y)$ is known, the shearing stress can be determined at all points by evaluating the velocity gradient, $du/dy$. For the distribution given

$$\frac{du}{dy} = -\frac{3Vy}{h^2} \qquad (2)$$

**(a)** Along the bottom wall $y = -h$ so that (from Eq. 2)

$$\frac{du}{dy} = \frac{3V}{h}$$

and therefore the shearing stress is

$$\tau_{\substack{\text{bottom}\\ \text{wall}}} = \mu\left(\frac{3V}{h}\right) = \frac{(0.04\ \text{lb} \cdot \text{s/ft}^2)(3)(2\ \text{ft/s})}{(0.2\ \text{in.})(1\ \text{ft}/12\ \text{in.})}$$

$$= 14.4\ \text{lb/ft}^2 \text{ (in direction of flow)} \qquad \text{(Ans)}$$

This stress creates a drag on the wall. Since the velocity distribution is symmetrical, the shearing stress along the upper wall would have the same magnitude and direction.

**(b)** Along the midplane where $y = 0$ it follows from Eq. 2 that

$$\frac{du}{dy} = 0$$

and thus the shearing stress is

$$\tau_{\text{midplane}} = 0 \qquad \text{(Ans)}$$

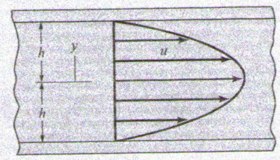



FIGURE E1.5a

**COMMENT** From Eq. 2 we see that the velocity gradient (and therefore the shearing stress) varies linearly with $y$ and in this particular example varies from 0 at the center of the channel to $14.4\ \text{lb/ft}^2$ at the walls. This is shown in Fig. E1.5*b*. For the more general case the actual variation will, of course, depend on the nature of the velocity distribution.

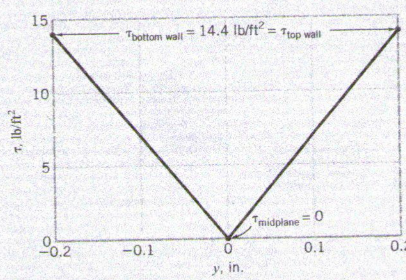



FIGURE E1.5b

**EXAMPLE 1.4** A machine creates small 0.5-mm-diameter bubbles of 20°C water. Estimate the pressure that exists inside the bubbles.

**Solution:** Bubbles have two surfaces leading to the following estimate of the pressure:

$$p = \frac{4\sigma}{r} = \frac{4 \times 0.0736}{0.0005} = 589\ \text{Pa}$$

where the surface tension was taken from Table C.1.

N O T E B O O K

## EXAMPLE 1.8 Capillary Rise in a Tube

**GIVEN** Pressures are sometimes determined by measuring the height of a column of liquid in a vertical tube.

**FIND** What diameter of clean glass tubing is required so that the rise of water at 20 °C in a tube due to capillary action (as opposed to pressure in the tube) is less than $h = 1.0$ mm?

### SOLUTION

From Eq. 1.22

$$h = \frac{2\sigma \cos\theta}{\gamma R}$$

so that

$$R = \frac{2\sigma \cos\theta}{\gamma h}$$

For water at 20 °C (from Table B.2), $\sigma = 0.0728$ N/m and $\gamma = 9.789$ kN/m$^3$. Since $\theta \approx 0°$ it follows that for $h = 1.0$ mm,

$$R = \frac{2(0.0728 \text{ N/m})(1)}{(9.789 \times 10^3 \text{ N/m}^3)(1.0 \text{ mm})(10^{-3} \text{ m/mm})} = 0.0149 \text{ m}$$

and the minimum required tube diameter, $D$, is

$$D = 2R = 0.0298 \text{ m} = 29.8 \text{ mm} \qquad \text{(Ans)}$$

**COMMENT** By repeating the calculations for various values of the capillary rise, $h$, the results shown in Fig. E1.8 are obtained.

Note that as the allowable capillary rise is decreased, the diameter of the tube must be significantly increased. There is always some capillarity effect, but it can be minimized by using a large enough diameter tube.

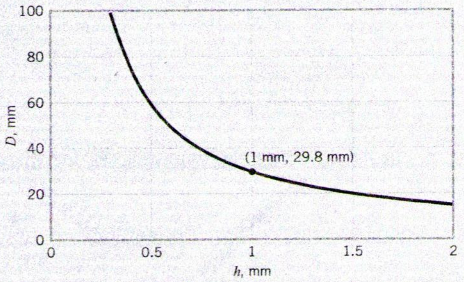
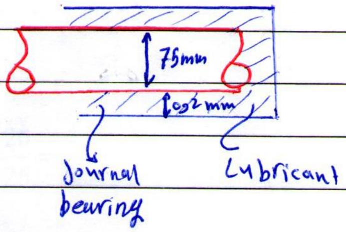



FIGURE E1.8

Subject *example :-* Date No.

problem :- The clearance between a 75 mm diameter and its journal bearing is 0,8 mm. The speed of the shaft is 120 rpm, find the shear stress induced in the Lubricant ; $\mu = 0,1D$ ?




Sol :-

$$\tau = \mu . \frac{du}{dy}$$

here, we can assume linear velocity variation (very small thickness)

$$\frac{du}{dy} = \frac{\Delta u}{\Delta y} = \frac{u}{\text{Clearance}}$$

$$u = 120 \text{ rpm} = 2 \text{ rps} = 2 * 2\pi * \frac{0,075}{2}$$

$$= 0,471 \text{ m/s}$$

$$\tau = 0,1 * \frac{0,471}{0,008} = 58,88 \text{ Pa}$$

N O T E B O O K

*problem:- what would be the minimum size of a glass tube to be selected be measure the water level, if it is desired that the capilary rise be limited to 0.25mm?

$\sigma_{s_{w}} = 0.073$ N/m ? $h = 0.25mm$ $\sigma_s = 0.073$

Find d ?

but for water $\phi = 0$, $\gamma = 9810$

$$d = \frac{4\sigma}{\gamma h} = \frac{4 \times 0.073}{9810 \times 0.025 \times 10^{-3}}$$

$d = 11.9$ cm

*problem:- what pressure increase must be applied to water to reduce its volume by 1% ? $E_v = 2.2$ GPa

$$E_v = \frac{-\Delta P}{\Delta \forall / \forall} \Rightarrow \Delta P = -E_v \frac{\Delta \forall}{\forall}$$

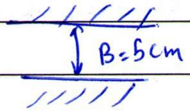
but $\frac{\Delta \forall}{\forall} = 1\%$ as given $\Rightarrow \frac{\Delta \forall}{\forall} = 0.01$

$\Rightarrow \Delta P = -2.2 \times 10^9 \times 0.01 = 22$ MPa

* problem:- Glycerin ... $T = 20°c$ pressure gradient $= \frac{dp}{dx} = 1.6$ kN/m³

what are the velocity and $\tau$ at $y = 12$mm from the wall?

$$u = \frac{-1}{2\mu} \cdot \frac{dp}{dx} [By - y^2]$$




$\mu = 6.2 \times 10^{-1}$

$$u = \frac{-1}{2 \times 6.2 \times 10^{-1}} \times (-1600) (0.05 \times 0.012 - 0.012^2)$$

$$\boxed{u = 0.588 \text{ m/s}}$$

Subject Date No.

**Example 1.6.** *The velocity distribution for flow over a plate is gives by $u = 2y - y^2$ where u is the velocity in m/s at a distance y metres above the plate. Determine the velocity gradient and shear stress at the boundary and 1.5 m from it.*

*Take dynamic viscosity of fluid as 0.9 N.s/m².*

$u = 2y - y^2 \Rightarrow \frac{du}{dy} = 2 - 2y$

Ⓐ Volcity gradient $\Rightarrow \frac{du}{dy}$

At $y = 0$, $\left.\frac{du}{dy}\right|_{y=0} = 2\ s^{-1}$

At $y = 0.15m$, $\left.\frac{du}{dy}\right|_{y=0.15} = 2 - 2 * 0.15 = 1.7\ s^{-1}$

Ⓑ Shear stress $\tau$:-

$\tau|_{y=0} = \mu \left.\frac{du}{dy}\right|_{y=0} = 0.9 * 2 = 1.8\ N/m^2$

$\tau|_{y=0.15} = \mu \left.\frac{du}{dy}\right|_{y=0.15} = 0.9 * 1.7\ N/m^2$

**Example 1.5.** *The space between two square flat parallel plates is filled with oil. Each side of the plate is 720 mm. The thickness of the oil film is 15 mm. The upper plate, which moves at 3 m/s requires a force of 120 N to maintain the speed. Determine:*

(i) *The dynamic viscosity of the oil;*

(ii) *The kinematic viscosity of oil if the specific gravity of oil is 0.95.*

$\boxed{s.g = \frac{\gamma_f}{\gamma_w} \quad \gamma_f = \rho g}$

Each side of square plate = 720 mm = 0.72m

The thickness oil = dy = 15mm = 0.015m

velocity of upper plate = 3m/s

$du = 3 - 0 = 3m/s$

$F = 120N$

$\tau = F/A = 120/(0.72 * 0.72) = 231.5\ N/m^3$

Ⓐ $\mu \Rightarrow \tau = \mu \frac{du}{dy} \Rightarrow 231.5 = \mu * \frac{3}{0.015}$

$\mu = 1.16\ N.s/m^2$

Ⓑ $\nu \Rightarrow W_{oil} = 0.95 * 9.81 = 9320\ N/m^3$

$\rho = W/g = 9320/9.81 = 950$ $\quad \nu = \frac{\mu}{\rho} = \frac{1.16}{950} = 0.00122\ m^2/s$

NOTEBOOK

Subject Date No.

**Example 1.11.** *A 150 mm diameter shaft rotates at 1500 r.p.m. in a 200 mm long journal bearing with 150.5 mm internal diameter. The uniform annular space between the shaft and the bearing is filled with oil of dynamic viscosity 0.8 poise. Calculate the power dissipated as heat.*
**(AMIE Winter, 2001)**

$d_{shaft} = 150mm$

$d_{bearing} = 150.5\,mm$

$L = 200\,mm = 0.2m$

$N = 1500\,rpm$

$\mu = 0.8$ poise $= 0.8 \times 0.1 = 0.08\,N.s/m^2$

Radial thickness of the oil $= dy = \dfrac{(150.5-150)/2}{1000}\,m = 0.00025\,m$

Tangential velocity of the shaft $u = \dfrac{\pi d N}{60} = \dfrac{\pi \times (150\times10^{-3})(1500)}{60}$

$\boxed{u = 11.78\,m/s}$

Change of velocity $du = u - 0 = 11.78\,m/s$

Tangential stress in oil $\tau = \mu \dfrac{du}{dy} = 0.08 \times \dfrac{11.78}{0.00025} = 3769.6\,N/m^2$

$P = Fw$

$= [\tau \cdot A]\,w = [\tau\,(\pi d L)]\,w$

$= 3769.6 \times \pi \times (150\times10^{-3}) \times 0.2 \times 11.78$

$= 4185\,W$

Subject Date No.

**Example 1.12.** *A vertical cylinder of diameter 180 mm rotates concentrically inside anothe. cylinder of diameter 181.2 mm. Both the cylinders are 300 mm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. Determine the viscosity of the fluid if a torque of 20 Nm is required to rotate the inner cylinder at 120 r.p.m.*

**Solution.** *Given*: Diameter of inner cylinder, $d = 180$ mm $= 0.18$ m

Diameter of outer cylinder, $D = 181.2$ mm $= 0.1812$ m

Length of each cylinder, $l = 300$ mm $= 0.3$ m

Speed of the inner cylinder, $N = 120$ r.p.m.

Torque, $T = 20$ Nm.

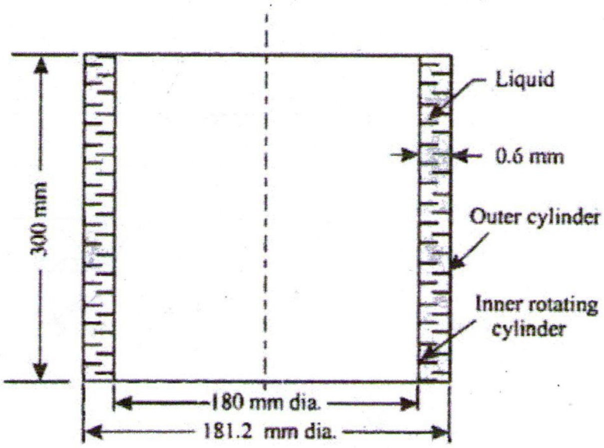



**Fig. 1.8**

$u = \frac{\pi d N}{60} = \frac{\pi * 0.18 * 120}{60} = 1.13\ m/s$

$A_{inner} = \pi d L = \pi * 0.18 * 0.3 = 0.1696\ m^2$

$\tau = \mu \frac{du}{dy}$

$du = u - 0 = 1.13 - 0 = 1.13\ m/s$

$dy = \frac{1.1812 - 0.18}{2}$ $= 0.0006\ m$

$\tau = 1883.33\ \mu$

$F = \tau * A = 1883.33\ \mu * 0.1696$

$T = F * \frac{d}{2} = 1883.33\ \mu * 0.1696 * \frac{0.18}{2} = 20$

$\boxed{\mu = 6.96\ \text{poise}}$ $\boxed{\mu = 0.696\ N.s/m^2}$

Subject Date No.

**Example 1.22.** *If the surface tension at air-water interface is 0.069 N/m, what is the pressure difference between inside and outside of an air bubble of diameter 0.009 mm?*

**Solution.** *Given*: $\sigma = 0.069$ N/m; $d = 0.009$ mm

*An air bubble has only one surface.* Hence, droplet $P = \frac{2\sigma}{r}$

$$p = \frac{4\sigma}{d}$$

$$= \frac{4 \times 0.069}{0.009 \times 10^{-3}} = 30667 \text{ N/m}^2$$

$$= \mathbf{30.667\ kN/m^2\ or\ kPa\ (Ans.)}$$

**Example 1.23.** *If the surface tension at the soap-air interface is 0.09 N/m, calculate the internal pressure in a soap bubble of 28 mm diameter.*

**Solution.** *Given*: $\sigma = 0.09$ N/m; $d = 28$ mm.

In a soap bubble there are two interfaces. Hence, $P = \frac{4\sigma}{r}$

$$p = \frac{8\sigma}{d} = \frac{8 \times 0.09}{28 \times 10^{-3}} = \mathbf{25.71\ N/m^2}$$ **(above atmospheric pressure) (Ans.)**

**Example 1.27.** *A clean tube of diameter 2.5 mm is immersed in a liquid with a coefficient of surface tension = 0.4 N/m. The angle of contact of the liquid with the glass can be assumed to be 135°. The density of the liquid = 13600 kg/m³.*

*What would be the level of the liquid in the tube relative to the free surface of the liquid inside the tube.*

**Solution.** *Given*: $d = 2.5$ mm ; $\sigma = 4$ N/m, $\theta = 135°$; $\rho = 13600$ kg/m³

**Level of the liquid in the tube, h:**

The liquid in the tube rises (or falls) due to capillarity. The capillary rise (or fall),

$$h = \frac{4\sigma \cos\theta}{wd} \quad \text{...(Eqn. 1.20)}$$

$$= \frac{4 \times 0.4 \times \cos 135°}{(9.81 \times 13600) \times 2.5 \times 10^{-3}} \quad (\because w = \rho g)$$

$$= -3.39 \times 10^{-3} \text{m or } -3.39 \text{ mm}$$

Negative sign indicates that there is a capillary **depression (fall) of 3.39 mm. (Ans.)**

**Example 1.28.** *Assuming that the interstices in a clay are of size equal to one tenth the mean diameter of the grain, estimate the height to which water will rise in a clay soil of average grain diameter of 0.048 mm. Assume surface tension at air-water interface as 0.074 N/m.*

**Solution.** *Given:* Diameter of the pores, $d = \frac{1}{10} \times 0.048 = 0.0048$ mm; $\sigma = 0.074$ N/m

Assuming $\theta = 0°$

$$h = \frac{4\sigma}{wd} = \frac{4 \times 0.074}{(9.81 \times 1000) \times 0.0048 \times 10^{-3}} = \mathbf{6.286\ m\ (Ans.)}$$

Subject Date No.

**Example 1.30.** *Determine the minimum size of glass tubing that can be used to measure water level, if the capillary rise in the tube is not to exceed 0.3 mm. Take surface tension of water in contact with air as 0.0735 N/m.*

**Solution.** *Given* :Capillary rise, $h = 0.3$ mm $= 0.3 \times 10^{-3}$ m

Surface tension, $\sigma = 0.0735$ N/m

Specific weight of water, $w = 9810$ N/m³.

**Size of glass tubing, $d$:**

Capillary rise, $$h = \frac{4\sigma\cos\theta}{wd} = \frac{4\sigma}{wd}$$

(Assuming $\theta = 0$ for water)

$$0.3 \times 10^{-3} = \frac{4 \times 0.0735}{9810 \times d}$$

$$\therefore \quad d = \frac{4 \times 0.0735}{0.3 \times 10^{-3} \times 9810} = 0.1 \text{ m} = \textbf{100 mm (Ans.)}$$

**Example 1.31.** *A U-tube is made up of two capillaries of bores 1.2 m and 2.4 mm respectively. The tube is held vertical and partially filled with liquid of surface tension 0.06 N/m and zero contact angle. If the estimated difference in the level of two menisci is 15 mm, determine the mass density of the liquid.*

**Solution.** *Given*: Bores of the capillaries:

$$d_1 = 1.2 \text{ mm} = 0.0012 \text{ m}$$
$$d_2 = 2.4 \text{ mm} = 0.0024 \text{ m}$$

Difference of level, $h_1 - h_2 = 15$ mm $= 0.015$ m; Angle of contact, $\theta = 0$

**Mass density of the liquid, $\rho$:**

$$h_1 = \frac{4\sigma\cos\theta}{\cancel{w}d_1}, \quad \text{and} \quad h_2 = \frac{4\sigma\cos\theta}{\cancel{w}d_2}$$

[where $w (= \rho g)$ = weight density of the liquid]

$$\therefore \quad h_1 - h_2 = \frac{4\sigma}{\cancel{w}}\left[\frac{1}{d_1} - \frac{1}{d_2}\right] \qquad (\because \theta = 0)$$

$$0.015 = \frac{4 \times 0.06}{\rho \times 9.81}\left[\frac{1}{0.0012} - \frac{1}{0.0024}\right] = \frac{0.02446}{\rho} \times 416.67$$

$$\therefore \quad \rho = \frac{0.02446 \times 416.67}{0.015} = \textbf{679.45 kg/m}^3 \textbf{ (Ans.)}$$

N O T E B O O K

Subject Date No.

**Problem 1.4** *A plate, 0.025 mm distant from a fixed plate, moves at 60 cm/s and requires a force of 2 N per unit area i.e., 2 N/m² to maintain this speed. Determine the fluid viscosity between the plates.*

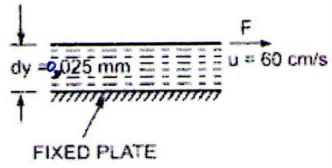



Fig. 1.3

distance between plates = $dy = 0.025\,mm$
$= 0.025 \times 10^{-3}\,m$

$u = 60\,cm/s = 0.6\,m/s$

$\tau = 2\,N/m^2$

$\tau = \mu \frac{du}{dy}$ $du = u - 0 = 0.6\,m/s$

$dy = 0.25 \times 10^{-3}\,m$

$2 = \mu \frac{0.6}{0.025 \times 10^{-3}} \Rightarrow \mu = \frac{2 \times 0.025 \times 10^{-3}}{0.6} = 8.33 \times 10^{-5}\,N.s/m^2$

$= 8.33 \times 10^{-4}$ poise.

**Problem 1.5** *A flat plate of area $1.5 \times 10^6$ mm² is pulled with a speed of 0.4 m/s relative to another plate located at a distance of 0.15 mm from it. Find the force and power required to maintain this speed, if the fluid separating them is having viscosity as 1 poise.*

**Solution.** Given :

Area of the plate, $A = 1.5 \times 10^6\,mm^2 = 1.5\,m^2$

Speed of plate relative to another plate, $du = 0.4$ m/s

Distance between the plates, $dy = 0.15\,mm = 0.15 \times 10^{-3}\,m$

Viscosity $\mu = 1 \text{ poise} = \frac{1}{10}\frac{Ns}{m^2}$.

Using equation (1.2) we have $\tau = \mu \frac{du}{dy} = \frac{1}{10} \times \frac{0.4}{.15 \times 10^{-3}} = 266.66 \frac{N}{m^2}$

(*i*) ∴ Shear force, $F = \tau \times \text{area} = 266.66 \times 1.5 =$ **400 N. Ans.**

(*ii*) Power* required to move the plate at the speed 0.4 m/sec

$= F \times u = 400 \times 0.4 =$ **160 W. Ans.**

Subject Date No.

**Problem 1.6** *Determine the intensity of shear of an oil having viscosity = 1 poise. The oil is used for lubricating the clearance between a shaft of diameter 10 cm and its journal bearing. The clearance is 1.5 mm and the shaft rotates at 150 r.p.m.*

$\mu = 1$ poise $= 0.1\ N.s/m^2$

$D = 10$ cm $= 0.1$ m

distance between shaft and journal bearing

$dy = 1.5\,mm = 1.5 \times 10^{-3}\,m$

speed shaft $N = 150$ rpm

Tangential speed shaft $u = \frac{\pi D N}{60} = \frac{\pi \times 0.1 \times 150}{60} = 0.785\ m/s$

$\tau = \mu \frac{du}{dy}$ $du = u - 0 = u$

$= \frac{1}{10} \times \frac{0.785}{1.5 \times 10^{-3}} = 52.33\ N/m^2$

Subject Date No.

**Problem 1.7** *Calculate the dynamic viscosity of an oil, which is used for lubrication between a square plate of size 0.8 m × 0.8 m and an inclined plane with angle of inclination 30° as shown in Fig. 1.4. The weight of the square plate is 300 N and it slides down the inclined plane with a uniform velocity of 0.3 m/s. The thickness of oil film is 1.5 mm.*

**Solution.** Given :

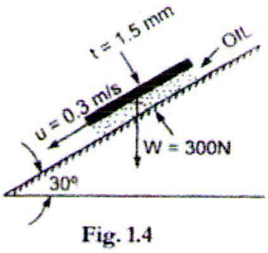



**Fig. 1.4**

Area of plate, $A = 0.8 \times 0.8 = 0.64\ m^2$

Angle of plane, $\theta = 30°$

Weight of plate, $W = 300$ N

Velocity of plate, $u = 0.3$ m/s

Thickness of oil film, $t = dy = 1.5\ mm = 1.5 \times 10^{-3}$ m

Let the viscosity of fluid between plate and inclined plane is $\mu$.

Component of weight $W$, along the plane $= W \cos 60° = 300 \cos 60° = 150$ N

Thus the shear force, $F$, on the bottom surface of the plate $= 150$ N

and shear stress, $$\tau = \frac{F}{\text{Area}} = \frac{150}{0.64}\ N/m^2$$

$W \sin 30 = \tau \cdot A$

Now using equation (1.2), we have

$$\tau = \mu \frac{du}{dy}$$

where $du$ = change of velocity $= u - 0 = u = 0.3$ m/s

$dy = t = 1.5 \times 10^{-3}$ m

$$\therefore \quad \frac{150}{0.64} = \mu \frac{0.3}{1.5 \times 10^{-3}}$$

$$\therefore \quad \mu = \frac{150 \times 1.5 \times 10^{-3}}{0.64 \times 0.3} = 1.17\ N\ s/m^2 = 1.17 \times 10 = \textbf{11.7 poise. Ans.}$$

N O T E B O O K

**Problem 1.9** *The space between two square flat parallel plates is filled with oil. Each side of the plate is 60 cm. The thickness of the oil film is 12.5 mm. The upper plate, which moves at 2.5 metre per sec requires a force of 98.1 N to maintain the speed. Determine :*

*(i) the dynamic viscosity of the oil in poise, and*

*(ii) the kinematic viscosity of the oil in stokes if the specific gravity of the oil is 0.95.*

(A.M.I.E., Winter 1977)

**Solution.** Given :

Each side of a square plate $= 60 \text{ cm} = 0.60 \text{ m}$

$\therefore$ Area, $A = 0.6 \times 0.6 = 0.36 \text{ m}^2$

Thickness of oil film, $dy = 12.5 \text{ mm} = 12.5 \times 10^{-3} \text{ m}$

Velocity of upper plate, $u = 2.5 \text{ m/sec}$

$\therefore$ Change of velocity between plates, $du = 2.5 \text{ m/sec}$

Force required on upper plate, $F = 98.1 \text{ N}$

$\therefore$ Shear stress, $\tau = \dfrac{\text{Force}}{\text{Area}} = \dfrac{F}{A} = \dfrac{98.1 \text{ N}}{0.36 \text{ m}^2}$

(i) Let $\mu$ = Dynamic viscosity of oil

Using equation (1.2), $\tau = \mu \dfrac{du}{dy}$ or $\dfrac{98.1}{0.36} = \mu \times \dfrac{2.5}{12.5 \times 10^{-3}}$

$$\therefore \quad \mu = \frac{98.1}{0.36} \times \frac{12.5 \times 10^{-3}}{2.5} = 1.3635 \frac{\text{Ns}}{\text{m}^2} \qquad \left(\because \frac{1 \text{ Ns}}{\text{m}^2} = 10 \text{ poise}\right)$$

$= 1.3635 \times 10 =$ **13.635 poise. Ans.**

(ii) Sp. gr. of oil, $S = 0.95$

Let $\nu$ = kinematic viscosity of oil

$\gamma_{oil} = S g_{oil} \times D_{water}$

$\gamma_{oil} = \rho_{oil} \cdot g$

Using equation (1.1A),

Mass density of oil, $\rho = S \times 1000 = 0.95 \times 1000 = 950 \text{ kg/m}^3$

$\rho_{oil} = 950 \text{ kg/m}^3$

Using the relation, $\nu = \dfrac{\mu}{\rho}$, we get $\nu = \dfrac{1.3635 \left(\dfrac{\text{Ns}}{\text{m}^2}\right)}{950} = .001435 \text{ m}^2\text{/sec} = .001435 \times 10^4 \text{ cm}^2\text{/s}$

= **14.35 stokes. Ans.** $(\because \text{cm}^2\text{/s} = \text{stoke})$

Subject Date No.

**Problem 1.11.** *Determine the specific gravity of a fluid having viscosity 0.05 poise and kinematic viscosity 0.035 stokes.*

**Solution.** Given :

Viscosity, $\mu = 0.05$ poise $= \frac{0.05}{10}$ N s/m$^2$

Kinematic viscosity, $\nu = 0.035$ stokes

$= 0.035$ cm$^2$/s $\{\because$ Stoke $=$ cm$^2$/s$\}$

$= 0.035 \times 10^{-4}$ m$^2$/s

Using the relation $\nu = \frac{\mu}{\rho}$, we get $0.035 \times 10^{-4} = \frac{0.05}{10} \times \frac{1}{\rho}$

$$\therefore \quad \rho = \frac{0.05}{10} \times \frac{1}{.035 \times 10^{-4}} = 1428.5 \text{ kg/m}^3$$

$$\therefore \quad \text{Sp. gr. of liquid} = \frac{\text{Density of liquid}}{\text{Density of water}} = \frac{1428.5}{1000} = 1.4285 \simeq \mathbf{1.43.\ Ans.}$$

**Problem 1.12** *Determine the viscosity of a liquid having kinematic viscosity 6 stokes and specific gravity 1.9.*

**Solution.** Given :

Kinematic viscosity $\nu = 6$ stokes $= 6$ cm$^2$/s $= 6 \times 10^{-4}$ m$^2$/s

Sp. gr. of liquid $= 1.9$

Let the viscosity of liquid $= \mu$

$$\text{Now sp. gr. of a liquid} = \frac{\text{Density of the liquid}}{\text{Density of water}}$$

$$\text{or} \quad 1.9 = \frac{\text{Density of liquid}}{1000}$$

$$\therefore \quad \text{Density of liquid} = 1000 \times 1.9 = 1900 \frac{\text{kg}}{\text{m}^3}$$

$\therefore$ Using the relation $\nu = \frac{\mu}{\rho}$, we get

$6 \times 10^{-4} = \mu / 1900$ $\mu = 1{,}14$ N.S/m$^2$

$\mu = 1{,}14 \times 10 = 11{,}4$ poise

Subject Date No.

**Problem 1.17** *A 15 cm diameter vertical cylinder rotates concentrically inside another cylinder of diameter 15.10 cm. Both cylinders are 25 cm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. If a torque of 12.0 Nm is required to rotate the inner cylinder at 100 r.p.m., determine the viscosity of the fluid.* (A.M.I.E., Winter, 1979)

**Solution.** Given :

Diameter of cylinder $= 15$ cm $= 0.15$ m

Dia. of outer cylinder $= 15.10$ cm $= 0.151$ m

Length of cylinders, $L = 25$ cm $= 0.25$ m

Torque, $T = 12.0$ Nm

Speed, $N = 100$ r.p.m.

Let the viscosity $= \mu$

Tangential velocity of cylinder, $u = \frac{\pi DN}{60} = \frac{\pi \times 0.15 \times 100}{60} = 0.7854$ m/s

Surface area of cylinder, $A = \pi D \times L = \pi \times 0.15 \times 0.25 = .1178 \text{ m}^2$

Now using relation $\tau = \mu \frac{du}{dy}$ ...(*i*)

where $du = u - 0 = u = .7854$ m/s

$$dy = \frac{0.151 - 0.150}{2} \text{ m} = .0005 \text{ m}$$

Substituting these values in equation (*i*), we get

$$\tau = \frac{\mu \times .7854}{.0005}$$

$\therefore$ Shear force, $F = \text{Shear stress} \times \text{Area} = \frac{\mu \times .7854}{.0005} \times .1178$

$\therefore$ Torque, $T = F \times \frac{D}{2}$ $12 = \mu * \frac{0.7854}{0.0005} * 0.1178 * \frac{0.15}{2}$

$\mu = 0.864 \text{ Ns/m}^2$

N O T E B O O K

Subject Date No.

**Problem 1.17A** *A vertical cylinder of diameter 180 mm rotates concentrically inside another cylinder of diameter 181.2 mm. Both the cylinders are 300 mm high. The space between the cylinders is filled with a liquid whose viscosity is unknown. Determine the viscosity of the fluid if a torque of 20 Nm is required to rotate the inner cylinder at 120 r.p.m.* (VTU, Aug. 2003)

**Solution.** Given:

Dia. of inner cylinder, $D_i$ = 180 mm = 0.180 m ;

Dia. of outer cylinder, $D_o$ = 181.2 mm = 0.1812 m ; height of cylinders, $h$ = 300 mm = 0.3 m ;

Torque, $T$ = 20 Nm ; speed, $N$ = 120 r.p.m.

Tangential velocity of rotating cylinder,

$$u = \frac{\pi DN}{60} = \frac{\pi D_i N}{60}$$

$$= \frac{\pi \times 0.180 \times 120}{60} = 1.13 \text{ m/s}$$

(Here $D = D_i$ as inner cylinder is rotating)

Surface area of rotating cylinder $= \pi D_i \times h$

$$= \pi \times 0.18 \times 0.3 = 0.1696 \text{ m}^2$$

Now using the relation, $\tau = \mu \frac{du}{dy}$

where $du = u - 0 = u = 1.13$ m/s ; $dy = \frac{D_o - D_i}{2}$

$$= \frac{0.1812 - 0.180}{2} = 0.0006 \text{ m, we get}$$

$$\tau = \mu \times \frac{1.13}{0.0006}$$

∴ Shear force, $F = \tau \times \text{Area} = \left(\frac{\mu \times 1.13}{0.0006}\right) \times 0.1696$

∴ Torque, $T = F \times \frac{D}{2} = \frac{F \times D_i}{2}$

or $$20 = \left(\frac{\mu \times 1.13}{0.0006} \times 0.1696\right) \times \frac{0.180}{2}$$

∴ $$\mu = \frac{20 \times 0.0006 \times 2}{1.13 \times 0.1696 \times 0.180} = 0.695 \text{ N s/m}^2$$

$= 0.695 \times 10 =$ **6.95 poise. Ans.**

Subject Date No.

**Problem 1.24** *What is the bulk modulus of elasticity of a liquid which is compressed in a cylinder from a volume of 0.0125 $m^3$ at 80 N/$cm^2$ pressure to a volume of 0.0124 $m^3$ at 150 N/$cm^2$ pressure ?*

**Solution.** Given :

Initial volume, $\forall = 0.0125 \text{ m}^3$

Final volume $= 0.0124 \text{ m}^3$

$\therefore$ Decrease in volume, $d\forall = .0125 - .0124 = .0001 \text{ m}^3$

$$\therefore \quad -\frac{d\forall}{\forall} = \frac{.0001}{.0125}$$

Initial pressure $= 80 \text{ N/cm}^2$

Final pressure $= 150 \text{ N/cm}^2$

$\therefore$ Increase in pressure, $dp = (150 - 80) = 70 \text{ N/cm}^2$

Bulk modulus is given by equation (1.10) as

$$K = \frac{dp}{-\frac{d\forall}{\forall}} = \frac{70}{\frac{.0001}{.0125}} = 70 \times 125 \text{ N/cm}^2$$

$$= \mathbf{8.75 \times 10^3 \ N/cm^2. \ Ans.}$$

**Problem 1.26** *Find the surface tension in a soap bubble of 40 mm diameter when the inside pressure is 2.5 N/$m^2$ above atmospheric pressure.*

**Solution.** Given :

Dia. of bubble, $d = 40 \text{ mm} = 40 \times 10^{-3} \text{ m}$

Pressure in excess of outside, $p = 2.5 \text{ N/m}^2$

For a soap bubble, using equation (1.15), we get

$$p = \frac{8\sigma}{d} \quad \text{or} \quad 2.5 = \frac{8 \times \sigma}{40 \times 10^{-3}}$$

$$\sigma = \frac{2.5 \times 40 \times 10^{-3}}{8} \text{ N/m} = \mathbf{0.0125 \ N/m. \ Ans.}$$

**Problem 1.27** *The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/$cm^2$ (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.*

**Solution.** Given :

Dia. of droplet, $d = 0.04 \text{ mm} = .04 \times 10^{-3} \text{ m}$

Pressure outside the droplet $= 10.32 \text{ N/cm}^2 = 10.32 \times 10^4 \text{ N/m}^2$

Surface tension, $\sigma = 0.0725 \text{ N/m}$

The pressure inside the droplet, in excess of outside pressure is given by equation (1.14)

or
$$p = \frac{4\sigma}{d} = \frac{4 \times 0.0725}{.04 \times 10^{-3}} = 7250 \text{ N/m}^2 = \frac{7250 \text{ N}}{10^4 \text{ cm}^2} = 0.725 \text{ N/cm}^2$$

Subject Date No.

**Problem 1.30.** *The capillary rise in the glass tube is not to exceed 0.2 mm of water. Determine its minimum size, given that surface tension for water in contact with air = 0.0725 N/m.*

**Solution.** Given :

Capillary rise, $h = 0.2 \text{ mm} = 0.2 \times 10^3 \text{ m}$

Surface tension, $\sigma = 0.0725 \text{ N/m}$

Let dia. of tube $= d$

The angle $\theta$ for water $= 0$

Density ($\rho$) for water $= 1000 \text{ kg/m}^3$

Using equation (1.20), we get

$$h = \frac{4\sigma}{\rho \times g \times d} \text{ or } 0.2 \times 10^{-3} = \frac{4 \times 0.0725}{1000 \times 9.81 \times d}$$

$$\therefore \quad d = \frac{4 \times 0.0725}{1000 \times 9.81 \times .2 \times 10^{-3}} = 0.148 \text{ m} = \mathbf{14.8 \text{ cm. Ans.}}$$

Thus minimum diameter of the tube should be 14.8 cm.

**Problem 1.31** *Find out the minimum size of glass tube that can be used to measure water level if the capillary rise in the tube is to be restricted to 2 mm. Consider surface tension of water in contact with air as 0.073575 N/m.* (Converted to SI Units, A.M.I.E., Summer, 1985)

**Solution.** Given :

Capillary rise, $h = 2.0 \text{ mm} = 2.0 \times 10^{-3} \text{ m}$

Surface tension, $\sigma = 0.073575 \text{ N/m}$

Let dia. of tube $= d$

The angle $\theta$ for water $= 0$

The density for water, $\rho = 1000 \text{ kg/m}^3$

Using equation (1.20), we get

$$h = \frac{4\sigma}{\rho \times g \times d} \text{ or } 2.0 \times 10^{-3} = \frac{4 \times 0.073575}{1000 \times 9.81 \times d}$$

N O T E B O O K

Subject Date No.

**Example 1.3 :** *If density of liquid is 837 kg/m$^3$, find its specific weight, specific gravity and specific volume. If kinematic viscosity of this liquid is 1.73 cm$^2$/s, obtain its dynamic viscosity* [1989, 2005]

**Solution :**

a) **Specific weight (w)** $= \rho \times g = 837 \times 9.81 = 8210.97\ \text{N/m}^3$

b) **Specific gravity** $= \dfrac{\text{Density of liquid}}{\text{Density of water}} = \dfrac{837}{1000} = 0.837$

c) **Specific volume** $= \dfrac{1}{w} = 1.218 \times 10^{-4}\ \text{m}^3/\text{N}$

or $= \dfrac{1}{\rho} = \dfrac{1}{837} = 1.19 \times 10^{-3}\ \text{m}^3/\text{kg}$

d) **Kinematic viscosity** $\nu = \dfrac{\mu}{\rho}$ $(\therefore\ \nu = 1.73\ \text{cm}^2/\text{s} = 1.73 \times 10^{-4}\ \text{m}^2/\text{s})$

$\therefore$ $\mu = 837 \times 1.73 \times 10^{-4}$

$= 0.1448\ \text{N-s/m}^2$

1.2 If the kinematic viscosity of benzene is $7.42 \times 10^{-3}$ stokes and its mass density is 860 kg/m$^3$, determine its dynamic viscosity in kg/m s.

$$\nu = 7.42 \times 10^{-3}\ \text{S}$$

$$= 7.42 \times 10^{-3} \times 10^{-4}\ \text{m}^2/\text{s} = 7.42 \times 10^{-7}\ \text{m}^2/\text{s}$$

$\therefore$ $$\mu = \rho\nu = 860 \times 7.42 \times 10^{-7}$$

$$= 6.381 \times 10^{-4}\ \text{kg/m s}$$

1.3 If the mass density of a fluid is 789 kg/m$^3$, determine its specific weight and specific volume.

$$\gamma = \rho g = 789 \times 9.806$$

$$= 7736.934\ \text{N/m}^3 \quad \text{or} \quad 7.737\ \text{kN/m}^3$$

$$\text{Specific volume } v = \frac{1}{\gamma} = \frac{1}{7.737} = 0.129\ \text{m}^3/\text{kN}$$

1.4 Determine the kinematic viscosity of air at 20°C if its dynamic viscosity is $1.85 \times 10^{-4}$ poise and its mass density is 1.208 kg/m$^3$.

$$\mu = 1.85 \times 10^{-4}\ \text{poise}$$

$$= 1.85 \times 10^{-5}\ \text{kg/m s}$$

$\therefore$ $$\nu = \frac{\mu}{\rho} = \frac{1.85 \times 10^{-5}}{1.208} = 1.531 \times 10^{-5}\ \text{m}^2/\text{s}$$

Subject Date No.

1.5 The space between two parallel horizontal plates kept 5 mm apart is filled with crude oil of dynamic viscosity 2.5 kg/m s. If the lower plate is stationary and upper plate is pulled with a velocity of 1.75 m/s, determine the shear stress on the lower plate.

$$\frac{du}{dy} = \frac{1.75}{5 \times 10^{-3}} = 0.35 \times 10^3 \text{ s}^{-1}$$

$$\tau = \mu \frac{du}{dy} = 2.5 \times 0.35 \times 10^3$$

$$= 875.0 \text{ N/m}^2$$

1.6 A 50 mm diameter and 0.10 m long cylindrical body slides vertically down in a 52 mm diameter cylindrical tube. The space between the cylindrical body and tube wall is filled with oil of dynamic viscosity 1.9 N s/m$^2$. Determine its velocity of fall if its weight is 16 N (Fig. 1.4).

Let $U$ be its terminal velocity of fall. Shear stress $\tau$ will be

$$\tau = \mu \frac{du}{dy} = 1.9 \times \frac{U}{1 \times 10^{-3}}$$

$$= 1.9 \times 10^3 \, U \text{ N/m}^2$$

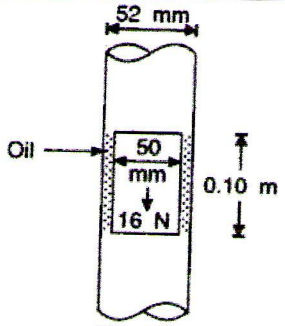


*Fig. 1.4.*

The shear stress will act on the surface of the cylinder. Hence

$$\text{Total force } F = \tau \times A$$

$$= 1.9 \times 10^3 \times U \times 3.142 \times 50 \times 10^{-3} \times 0.10$$

$$= 29.849 \, U$$

Under equilibrium condition, the weight will be balanced by the total shear force. Hence

$$16.0 = 29.849 \, U$$

$$U = 0.536 \text{ m/s}$$

Subject Date No.

1.7 A rectangular plate of 0.50 m × 0.50 m dimensions weighing 500 N slides down an inclined plane making 30° angle with horizontal, at a velocity of 1.75 m/s. If the 2 mm gap between the plate and inclined surface is filled with a lubricating oil, find its viscosity and express it in poise as well as in N s/m$^2$ (Fig. 1.5).

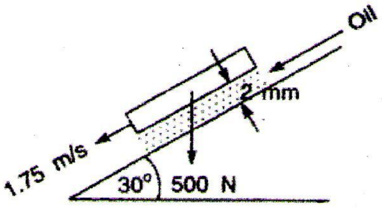



*Fig. 1.5.*

$$F = 0.5 \times 0.5 \times \mu \times \frac{1.75}{2 \times 10^{-3}}$$

$$= 0.219 \times 10^3 \mu$$

For equilibrium, this must be equal to component of weight along the inclined surface.

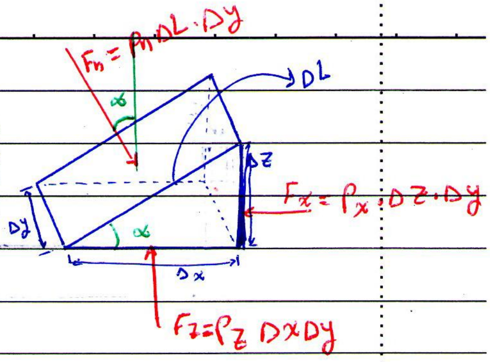
$$500 \sin 30° = 0.219 \times 10^3 \mu$$

$$\mu = \frac{500 * 0.5}{0.219} * 10^{-3} = 1.142 \ N.s/m^2$$

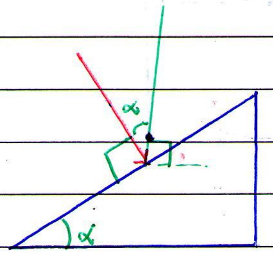
N O T E B O O K

Subject: chapter 3:- fluid statics    Date    No.

* pressure on point:-




$$P = \frac{F_n}{A}$$




$\Sigma F_x = 0$

$$P_n \Delta L \Delta y \sin\alpha - P_x \Delta z \Delta y = 0$$

$$\sin\alpha = \frac{\Delta z}{\Delta L}$$

$$P_n \cancel{\Delta L} \Delta y \frac{\Delta z}{\cancel{\Delta L}} - P_x \Delta z \Delta y = 0$$

$$P_n - P_x = 0 \Rightarrow \boxed{P_n = P_x}$$

$$P_z \Delta x \Delta y - P_n \Delta L \Delta y \cos\alpha = 0$$

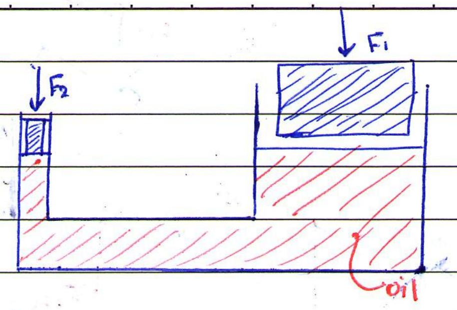
$$\cos\alpha = \frac{\Delta x}{\Delta L}$$

$$P_z \Delta x \Delta y - P_n \cancel{\Delta L} \Delta y \frac{\Delta x}{\cancel{\Delta L}} = 0$$

$$P_z - P_n = 0 \Rightarrow \boxed{P_z = P_n}$$

$\boxed{P_z = P_x = P_n}$ ⟹ الضغط عند نقطة له نفس القيمة بجميع الاتجاهات

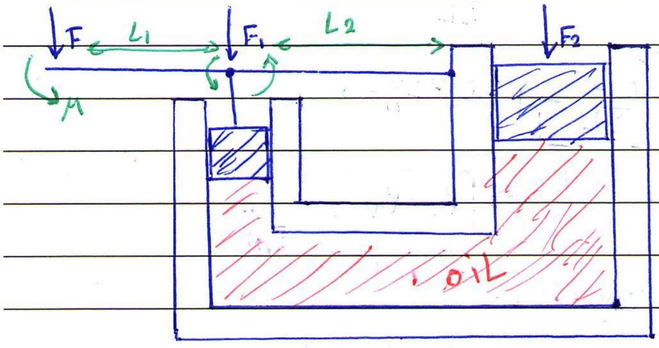
Subject Date No.




$$P_1 = P_2$$

$$\Rightarrow \frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow \boxed{F_1 = \frac{A_1}{A_2} \cdot F_2}$$

$F_1 > F_2$




$$F(L_1+L_2) = F_1 L_2$$

$$F_1 = \frac{F(L_1+L_2)}{L_2}$$

* $P_1 = P_2$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_2 = \frac{A_2}{A_1} F_1$$

$$\boxed{F_2 = \frac{F(L_1+L_2)}{L_2} \cdot \frac{A_2}{A_1}} \Leftarrow$$ تضعيف القوة بالذراع + المساحة

* absolute fluid :-

$$P = \frac{F}{A} \left[\frac{N}{m^2}\right] = [pa]$$

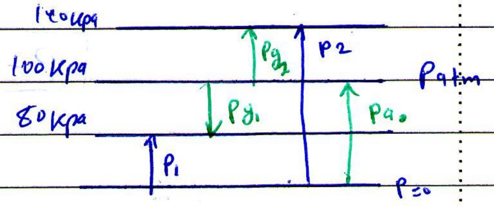
$1\,bar = 100\,kpa = 1 \times 10^5\,pa$

~~1 atm = 100 kpa = 1×10⁵ pa~~

$1\,atm = 101{,}325\,kpa$ الضغط الجوي بنسبة الى الضغط المعياري

$m_{H_2O}$, $mm_{H_2O}$, $cm_{Hg}$ } طول المادة ⇒ تحول الى pa ⇒ $P = \gamma_f\, h\,[pa]$ , $h \rightarrow [m]$

يختلف الضغط الجوي مع ← الظروف الجوية ، الارتفاع عن منطقة البحر ، درجات الحرارة .




$P_{g_1} = -20\,kpa$

$P_{g_2} = 40\,kpa$

$P_1, P_2$ = Absolute pressure

$P_{g_1}, P_{g_2}$ = P gauge

الضغط المطلق $P_{abs} = P_{atm} + P_{gauge}$ (الضغط الجوي ، الضغط المقاس)

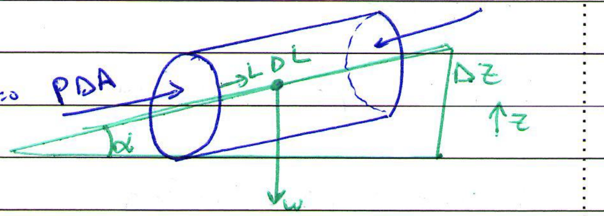
$P_1 = 100 + (-20) = 80\,kpa$

$P_2 = 100 + (40) = 140\,kpa$

Subject Date No.

* Pressure Variation with Elevation :-

تغير الضغط مع الارتفاع




$$P\Delta A - (P+\Delta P)\Delta A - W\sin\alpha = 0$$

$W \rightarrow \rho V g$

$$-\Delta p \cancel{\Delta A} - \rho(\cancel{\Delta A}\cancel{\Delta L})g\left(\frac{\Delta Z}{\cancel{\Delta L}}\right) = 0$$

$$-\Delta p - \gamma \Delta Z = 0 \Rightarrow -\Delta P = \gamma \Delta Z$$

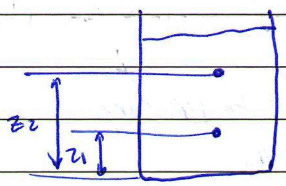
$$\boxed{\frac{\Delta P}{\Delta Z} = -\gamma} \Rightarrow \frac{dP}{dz} = -\gamma$$

$$\frac{dp}{dz} = -\gamma$$

$$dp = -\gamma\, dz$$

$$\boxed{p = -\gamma z + c}$$

$$\boxed{p + \gamma z = c}$$

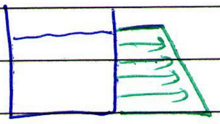



$$p_1 + \gamma z_1 = p_2 + \gamma z_2$$

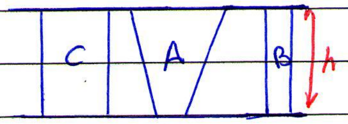
N O T E B O O K

Subject Date No.

* pressure measarment :~

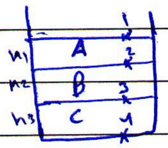


$\frac{dp}{dz} = -\gamma$ , $P = -\gamma z + c$




الضغط متساوي
لأن h متساوي

المائع الأكثر كثافة C




$P_1 = P_a$

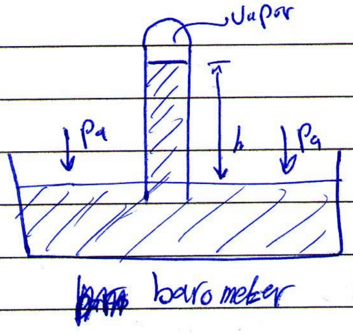
$P_2 = P_1 + \gamma_A h_1$

$P_3 = P_2 + \gamma_B h_2$

$P_4 = P_3 + \gamma_C h_3$

$P_1 + \gamma_A h_1 + \gamma_B h_2 + \gamma_C h_3 = P_4$

$P_4 - \gamma_C h_3 = P_3$



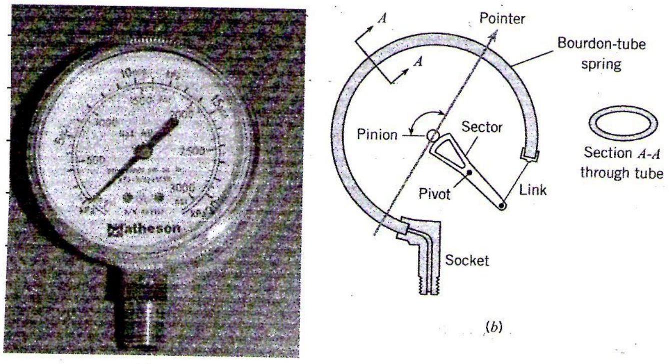

barometer

$\boxed{\gamma_{Hg}\, h = P_a}$

البارومتر يستخدم
لقياس الضغط الجوي

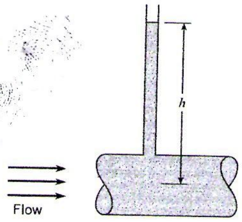
NOTEBOOK

Subject Date No.



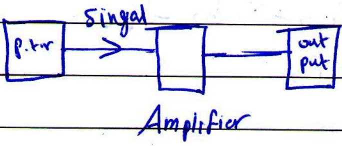

(a) (b)

⇒ bourdan gauge

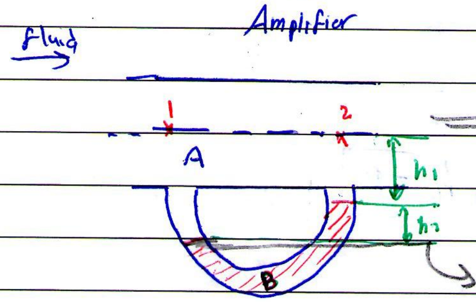



⇒ pizometer لقياس ضغط نظام

$P_a + \gamma_F h = P$ دائماً إيجابي




⇒ Pressure tronducter




⇒ manometer لقياس الضغط بين نقطتين أو نقطة

خط تساوي الضغوط (نفس مقدار الارتفاع)

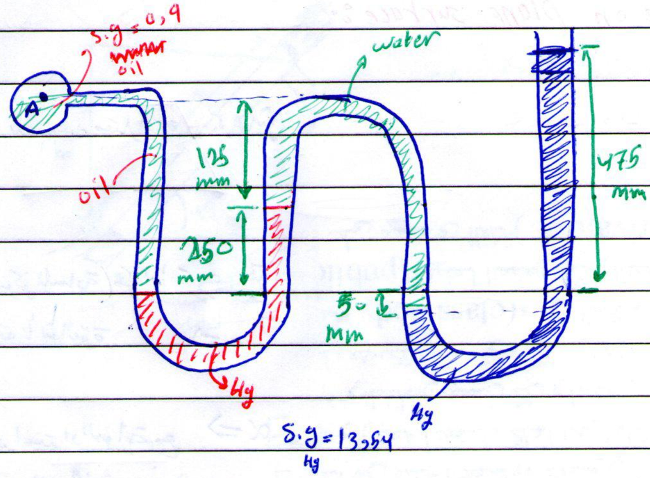
$P_1 + \gamma_A (h_1 + h_2) - \gamma_B h_2 - \gamma_A h_1 = P_2$

$P_1 - P_2 = \gamma_B h_2 + \cancel{\gamma_A h_1} - \cancel{\gamma_A h_1} = \gamma_B h_2 - \gamma_A h_2$

$= (\gamma_B - \gamma_A) * h_2$

Subject Date No.

ex:-




find $P_A$ :-

$$P_A + \gamma_{oil} \times 0{,}375 - 0{,}25\,\gamma_{Hg} + 0{,}3\,\gamma_w - 0{,}475\,\gamma_{Hg} = P_a$$

$\gamma_{Hg} \to 13{,}54\,\gamma_w$

$$(P_A - P_a) = 0{,}25 \times (13{,}54\,\gamma_w) + (0{,}75\,(13{,}54\,\gamma_w)) - 0{,}375\,(0{,}9\,\gamma_w) - 0{,}3\,\gamma_w$$

$$\gamma_w = \rho g = 1000 \times 9{,}81 = 9810\ N/m^2$$

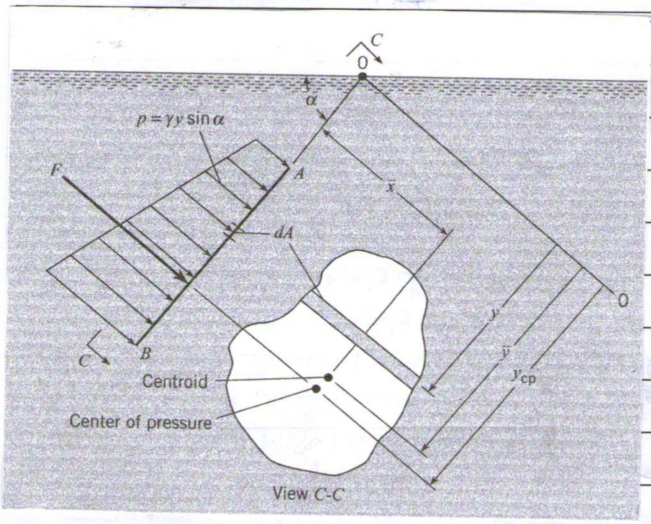
$$(P_A - P_a) = 90045{,}9\ Pa = 90{,}0459\ kPa$$

OR:-

$$P_a + \gamma_{Hg}\,0{,}475 - 0{,}3\,\gamma_w + 0{,}25\,\gamma_{Hg} - \gamma_{oil} \times 0{,}325 = P_A$$

Subject  Date  No.

## * hydrostatic force on plane surface :-




* نبحث عن مقدار القوة و مركز التأثير

$\bar{y} \rightarrow$ (مركز المساحة/ مركز البوابة)

$y_{cp} \Rightarrow$ موقع تأثير القوة

$\alpha \Rightarrow$ التقاء امتداد البوابة مع سطح السائل

$$dF = p\,dA = \gamma h\,dA$$

$$\int dF = \int \gamma y \sin\alpha \, dA$$

$$F = \gamma \sin\alpha \int y \, dA$$

$$\boxed{F = \gamma A \bar{y} \sin\alpha}$$

* $\sin\alpha = \frac{h}{y}$

$h = y \sin\alpha$

* $\bar{y} = \frac{1}{A} \int y \, dA$

$A\bar{y} = \int y \, dA$

M القوة المركزة حول O

$$F\, y_{cp} = \int \overset{\text{الذراع}}{y}\, \overset{\text{القوة}}{\overbrace{p\, dA}}$$

$$= \int y (\gamma y \sin\alpha)\, dA$$

$$F\, y_{cp} = \gamma \sin\alpha \int y^2 \, dA$$

$$= \gamma \sin\alpha \left[ I + \bar{y}^2 A \right]$$

$$\cancel{\gamma} A \bar{y} \cancel{\sin\alpha}\, y_{cp} = \cancel{\gamma \sin\alpha} \left[ I + \bar{y}^2 . A \right]$$

* نظرية المحاور المتوازية

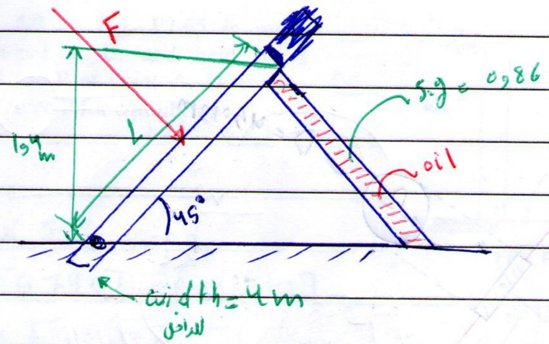
$\int y^2 \, dA$ يمثل M حول O

$= \underset{\text{المحور حول centroid}}{I} + \underset{\text{centroid}}{\bar{y} A}$ (ينقل M من المركز الى O)

$$\boxed{y_{cp} = \bar{y} + \frac{I}{\bar{y} . A}}$$

Subject Date No.

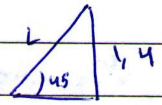
ex:-




$\bar{y} = \frac{L}{2}$ مركز البوابة

$L = \frac{1,4}{\cos 45}$

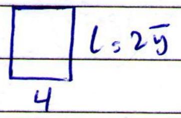
$\bar{y} = \left(\frac{1,4}{\cos 45}\right) * \frac{1}{2}$




$\bar{y} = 0,99\,m$

$A = L * w$

$= 2 * 0,99 * 4$




$F = \gamma A \bar{y} \sin\alpha$

$= 0,86 * 9810 * [4 * 0,99 * 2] * 0,99 \sin 45 = 46773\,N$

$= 46,773\,kN$

$y_{cp} = \bar{y} + \frac{I}{A\bar{y}}$

$= \bar{y} + \frac{\frac{1}{12} * 4 * (2\bar{y})^3}{[2\bar{y} * 4] * \bar{y}}$

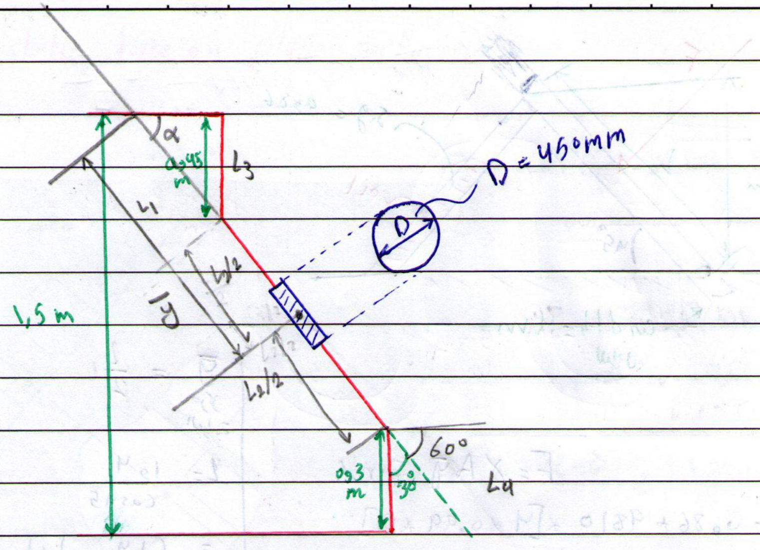
$= \bar{y} + \frac{\bar{y}}{3}$

$= 1,33 * 0,99 \Rightarrow y_{cp} = 1,32\,m$

Poletechnic Lecture Note

Subject                                    Date                    No.

ex:-

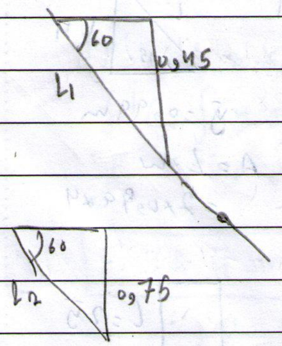



$\alpha = 60°$

ع امتداد افقي لسطح العمودي

$L_5 + L_4 + L_3 = 1.5$

$L_5 = 0.75\,m$




$$\bar{y} = L_1 + \frac{L_2}{2}$$

$$= \frac{0.45}{\sin 60} + \frac{1}{2}\left[\frac{0.75}{\sin 60}\right] = 0.952\,m$$

$$F = \gamma \bar{y} A \sin\alpha$$

$$= [0.85 * 9810] * 0.952 * \left[\frac{\pi}{4}[0.45]^2\right] * \sin 60 = 1094\,N$$

$$= 1.094\,kN$$

$$y_{cp} = \bar{y} + \frac{I}{A\bar{y}} = 0.952 + \frac{\frac{\pi}{64}[0.45]^4}{\frac{\pi}{4} * [0.45]^2 * 0.952}$$

$$I = \frac{\pi}{64} D^4$$

دائماً يأتي تحت السنترويد

$$= 0.965$$

N O T E B O O K

Subject Date No.

**2.65** Gate AB in Fig. P2.65 is semicircular, hinged at B, and held by a horizontal force P at point A. Determine the required force P for equilibrium.

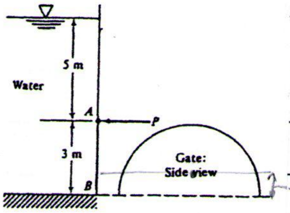



Fig. P2.65

$\gamma = 9790$

$F = \gamma \bar{y} A \sin\alpha$

$= [9790][5 + 1,727]$

$* [14,137]$

$= 931 * 10^3 N$

$\bar{y} = \frac{4}{3\pi} r = 1,273 \text{ m}$

$3 - 1,273 = \boxed{1,727 \text{ m}}$

$A = \frac{\pi r^2}{2} = \frac{\pi * 3^2}{2} = 14,137$

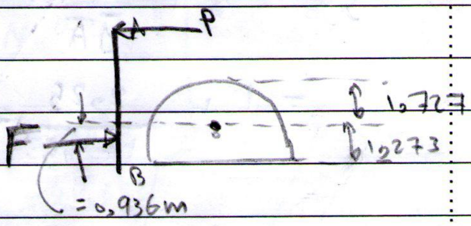
$y_{cp} = \bar{y} + \frac{\bar{I}}{\bar{y} A}$

$= [6,727] + \frac{[8,91]}{[6,727][14,137]}$

$\bar{I} = 0,11 * r^4 = 8,91$

$= 6,82$

$y_{cp} - \bar{y} = 0,0936$




$\Sigma M_B = 0$

$\Rightarrow (931 \times 10^3)(1,273 - 0,936) - 3P = 0$

$\frac{931 \times 10^3 * 0,337}{3} = P$

$P = 366 \times 10^3 N$

$= 366 kN$

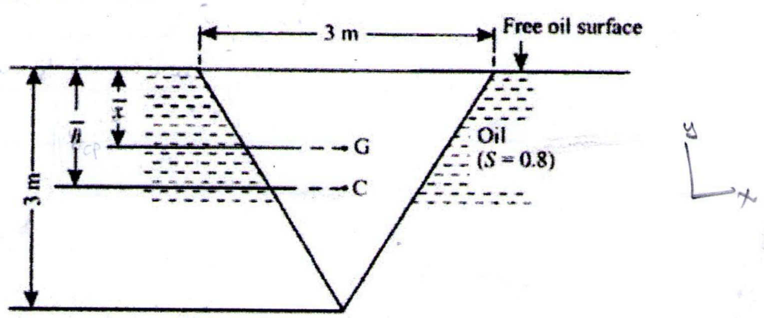
Subject Date No.

**Example 3.3.** *An isosceles triangular plate of base 3 m and altitude 3 m is immersed vertically in an oil of specific gravity 0.8. The base of the plate coincides with the free surface of oil. Determine:*

(*i*) *Total pressure on the plate;* (*ii*) *Centre of pressure.*

**Solution.** Base of the plate, $b = 3$ m

Height of the plate, $h = 3$ m




(A) $P = \gamma A \bar{y}$

$= (0.8 * 9810) * 4.5 * 1$

$= 35316\ N = 35.31\ kN$

$A = \frac{1}{2} \times 3 * 3 = 4.5\ m^2$

$\bar{y} = \frac{h}{3} = 1\ m$

(B) $h_p = \bar{x} + \frac{\bar{I}}{\bar{y} A}$

$= 1 + \frac{2.25}{1 * 4.5} = \boxed{1.5\ m}$

$\bar{I} = \frac{bh^3}{36}$

$= \frac{3 \times 3^3}{36} = 2.25$

Subject Date No.

**EXAMPLE 3.9 HYDROSTATIC FORCE DUE TO CONCRETE**

Determine the force acting on one side of a concrete form 2.44 m high and 1.22 m wide (8 ft by 4 ft) that is used for pouring a basement wall. The specific weight of concrete is 23.6 kN/m³ (150 lbf/ft³).

$\gamma = 23.6 \ kN/m^3$

Find the resultant force acting the wall ?

$F = \gamma \bar{y} A$

$= [23.6 kN/m^3][1.22 m]$

$[2.977 m^2]$

$= 85.7 kN$

$\bar{y} = \frac{2.44}{2} = 1.22$

$A = (2.44)(1.22)$

$= 2.977$

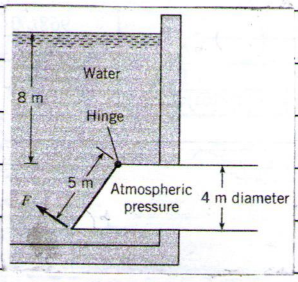
**EXAMPLE 3.10 FORCE TO OPEN AN ELLIPTICAL GATE**

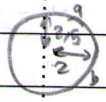
An elliptical gate covers the end of a pipe 4 m in diameter. If the gate is hinged at the top, what normal force $F$ is required to open the gate when water is 8 m deep above the top of the pipe and the pipe is open to the atmosphere on the other side? Neglect the weight of the gate.

*Problem Definition*

**Situation:** Water pressure is acting on an elliptical gate.

**Find:** Normal force (in newtons) required to open gate.




$z_{centroid} = 8 + 2 = 10$

$A = \pi ab = 2.5 * 2 * \pi$

$= 15.71 m^2$

$F = \gamma \bar{y} A = [9810][10][15.71]$

$= 1.54 MN$

depth pressure $\bar{z}$

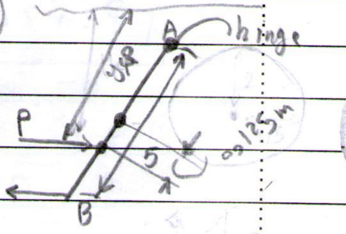
$y_{CB} = \bar{y} + \frac{\bar{I}}{\bar{y} A}$

$\bar{I} = \frac{\pi a^3 b}{4}$

$= 24.543$

$y_{CB} - \bar{y} = \frac{24.543}{[12.5][15.71]}$ $m^4$

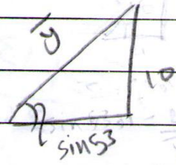
$y = 0.125 m$




$\Sigma M_{hinge} = 0$

$\Rightarrow [1.54 \times 10^6][2.625] - 5F = 0$

$\boxed{F = 809 kN}$




$\bar{y} = 12.5$

Subject Date No.

**2.68** Isosceles triangle gate AB in Fig. P2.68 is hinged at A and weighs 1500 N. What horizontal force P is required at point B for equilibrium?

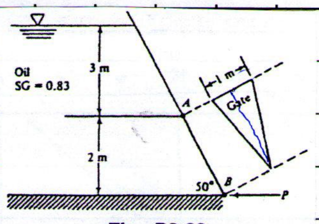
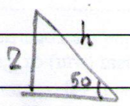



Fig. P2.68

$$F = \gamma A \bar{y} \sin\theta$$
$$= (0.83 * 9790)[1.3054] * [4.77][\sin 50]$$
$$= 38894 N$$

$$A = \frac{1}{2} bh = 1.3054$$



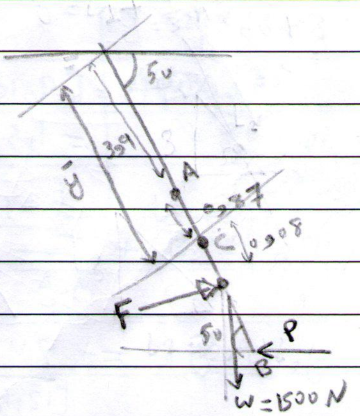

$$\sin 50 = \frac{2}{h}$$
$$h = 2.6$$

$$y_{CP} = \bar{y} + \frac{I}{\bar{y}A} = 4.77 + \frac{0.488}{4.77 * 1.3054}$$
$$= 4.85$$

$$y_{CP} - \bar{y} = 0.08$$

$$\Sigma M_A = 0 \Rightarrow -P(2) + (38894)(0.955) - 1500(0.559) = 0$$

$$\boxed{P = 18055.4 N}$$



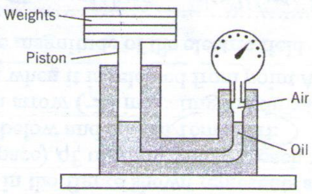

$$I = \frac{bh^3}{36} = 0.488$$

3, $L_2$, 50

$$L_2 = \frac{3}{\sin 50} = 3.9$$

Subject Problem Date No.

**3.4** The Crosby gage tester shown in the figure is used to calibrate or to test pressure gages. When the weights and the piston together weigh 140 N, the gage being tested indicates 200 kPa. If the piston diameter is 30 mm, what percentage of error exists in the gage?




PROBLEM 3.4

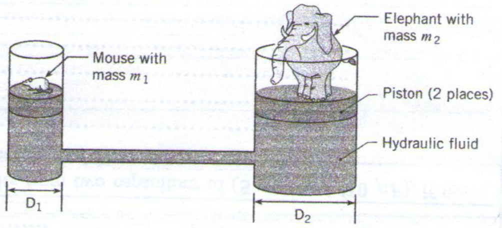
$$P_{true} = \frac{F}{A} = \frac{140\,N}{(\pi/4) * 0.03^2}$$

$$= 198.049\,kPa$$

$$\%\,error = \frac{P_{recorded} - P_{true}}{P_{true}} * 100\%$$

$$= \frac{(200-198)*100}{198} = 1.01\%$$

A mouse can have a mass of 25 g and an elephant a mass of 7500 kg. Determine a value of $D_1$ and $D_2$ so that the mouse can support the elephant.




PROBLEM 3.5

$m_m = 25\,g$ $m_{ele} = 7500\,kg$

$D_1 = ?$ $D_2 = ??$

$$P_1 = P_2$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

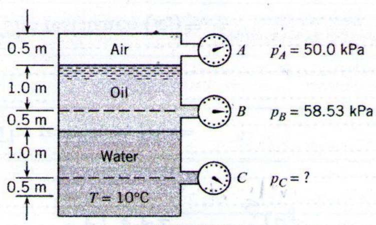
$$\frac{m_1 g}{\frac{\pi}{4} * D_1^2} = \frac{m_2 g}{\frac{\pi}{4} * D_2^2}$$

$$\frac{\pi}{4} m_1 D_2^2 = \frac{\pi}{4} m_2 \pi D_1^2$$

$$7500\,D_2 = 0.025\,D_1$$

$$\boxed{D_1 = 3 * 10^5 D_2}$$

**3.11** For the closed tank with Bourdon-tube gages tapped into it, what is the specific gravity of the oil and the pressure reading on gage *C*?




PROBLEM 3.11

$P_A + \gamma h_A = P_B + \gamma h_B$

$50\times10^3\ N/m^2 + \gamma_{oil}(1m)$

$= 58{,}530\ N/m^2 + \gamma_{oil}(0m)$

$\gamma_{oil} = 8530\ N/m^2$

$P_C = P_B + \gamma_w(1) + \gamma_{oil}(0.5)$

$s.g_{oil} = \frac{\gamma_{oil}}{\gamma_{water}} = \frac{8530}{9810} = 0.87$

$P_B = P_A + \gamma_{oil}(1)$

$\Rightarrow \gamma_{oil}$

$P_C = P_{btm\ of\ oil} + \gamma_w(1m)$

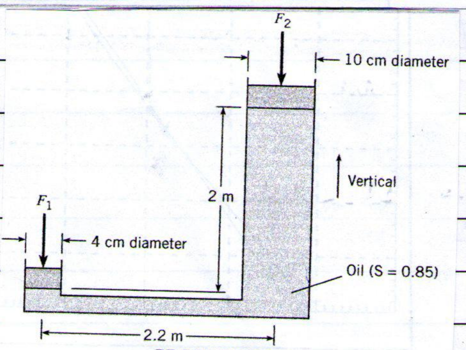
$9810(1)$

$P_{btm\ of\ oil} = 58{,}53\ Pa + \gamma_{oil}\times(0.5\ m)$

$P_C = 72.6\ kPa$

★

**3.13** If a 200 N force $F_1$ is applied to the piston with the 4 cm diameter, what is the magnitude of the force $F_2$ that can be resisted by the piston with the 10 cm diameter? Neglect the weights of the pistons.




PROBLEM 3.13

$F_1 = P_1A_1 \Rightarrow P_1 = \frac{F_1}{A_1} = \frac{4\times200N}{\frac{\pi}{4}\times0.04^2} = 1.592\times10^5 Pa$

hydrostatic eq:

$P_2 + \gamma z_2 = P_1 + \gamma z_1$

$P_2 = P_1 + (S\cdot\gamma_w)(z_1 - z_2)$

$= 1.592\times10^5 + (0.85\times9810)(-2)$

$= 1.425\times10^5\ Pa$

$F_2 = P_2A_2 = 1.425\times10^5 \times \frac{\pi}{4}\ 0.1^2$

$\boxed{F_2 = 1120\ N}$

$P_2 + \gamma_{oil}(2) = P_1$

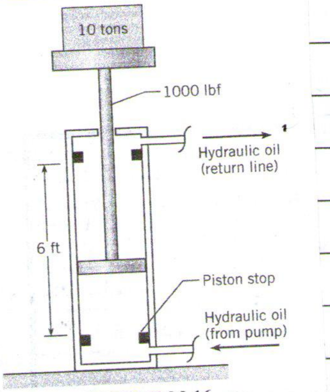
Subject Date No.

**3.14** Some skin divers go as deep as 50 m. What is the gage pressure at this depth in fresh water, and what is the ratio of the absolute pressure at this depth to normal atmospheric pressure? Assume $T = 20°C$.

$P = \gamma \Delta h = 9790 \times 50$
$= 489500\ N/m^2$
$= 489.5\ kPa$ gage.

Ratio $\frac{P_{50}}{P_{atm}} = \frac{489.5 + 101.3}{101.3} = 5.83$

**3.16** An engineer is designing a hydraulic lift with a capacity of 10 tons. The moving parts of this lift weigh 1000 lbf. The lift should raise the load to a height of 6 ft in 20 seconds. This will be accomplished with a hydraulic pump that delivers fluid to a cylinder. Hydraulic cylinders with a stroke of 72 inches are available with bore sizes from 2 to 8 inches. Hydraulic piston pumps with an operating pressure range from 200 to 3000 psig are available with pumping capacities of 5, 10, and 15 gallons per minute. Select a hydraulic pump size and a hydraulic cylinder size that can be used for this application.



PROBLEM 3.16

hydraulic lift
- capacity = 20,000 lbf (10 tons)
- weight of lift = 1000 lbf
- lift speed = 6 feet in 20 sec
- D = 2 to 8 inches

Piston pump { $P_{range}$ = 200 to 3000 psig & capacity = 5, 10, 15 gpm

(A) select hydraulic pump capacity :-

$A = \frac{F}{P_{max}} = \frac{21000}{3000} = 7\ in^2$

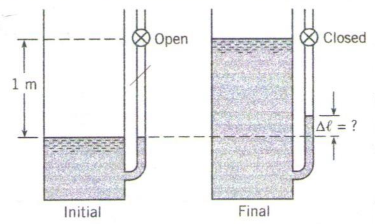
(B) cyliner diameter

$A = \frac{\pi}{4} D^2 \Rightarrow D = \sqrt{\frac{4}{\pi} A}$

$D_{min} = 2.98\ in$

Subject Date No.

**3.17** A tank with an attached manometer contains water at 20°C. The atmospheric pressure is 100 kPa. There is a stopcock located 1 m from the surface of the water in the manometer. The stopcock is closed, trapping the air in the manometer, and water is added to the tank to the level of the stopcock. Find the increase in elevation of the water in the manometer assuming the air in the manometer is compressed isothermally.




PROBLEM 3.17

$\gamma_w = 9790\ N/m^3$

$P V_1 = P_2 V_2$

$P_1 = 100{,}000\ N/m^2\ abs$

$V_1 = 1m * A_{tube}$

$P_2 = 100.000\ N/m^2 + \gamma_w(1m - \Delta L)$

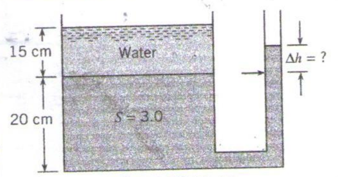
$V_2 = (1m - \Delta L)A_{tube}$

$[100{,}000][A\ \cancel{tube}] = [100{,}000 + \gamma_w[1m - \Delta L]]\ [1m - \Delta L]A\ \cancel{tube}$

$100.000 = (100{,}000 + 9810(1 - \Delta L))(1 - \Delta L)$

$\boxed{\Delta L = 0{,}0826\ m}$

**3.18** A tank is fitted with a manometer on the side, as shown. The liquid in the bottom of the tank and in the manometer has a specific gravity (S) of 3.0. The depth of this bottom liquid is 20 cm. A 15 cm layer of water lies on top of the bottom liquid. Find the position of the liquid surface in the manometer.




PROBLEM 3.18

$\frac{P_1}{\gamma_w} + z_1 = \frac{P_2}{\gamma_w} + z_2$

$P_1 + \gamma_w(0{,}15) = P_2$

$\frac{0\ Pa}{9810} + 0{,}15\ m = \frac{P_2}{9810} + 0\ m \Rightarrow P_2 = 1471{,}5\ Pa$

$\frac{P_2}{\gamma_{manometer\ fluid}} + z_2 = \frac{P_3}{\gamma_{mano.fluid}} + z_3$ $\quad P_1 + \gamma_w(0{,}15) - \gamma_f \Delta h = 0$

$\frac{1471{,}5}{3(9810)} + 0 = \frac{0}{\gamma_{mano}} + \Delta h \Longrightarrow \Delta h = 5cm$

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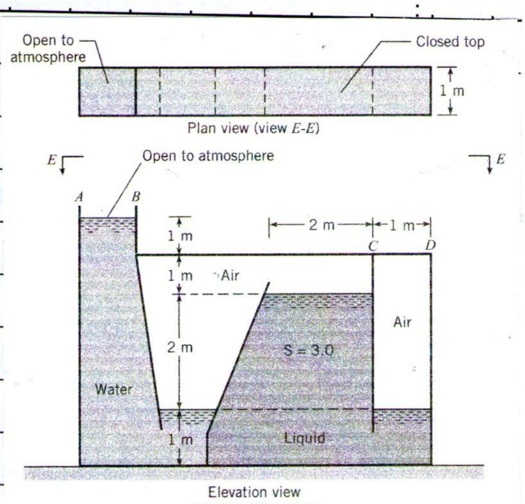
**3.21** What is the maximum gage pressure in the odd tank shown in the figure? Where will the maximum pressure occur? What is the hydrostatic force acting on the top (*CD*) of the last chamber on the right-hand side of the tank? Assume $T = 10°C$.




PROBLEM 3.21

$$0 + 4\times\gamma_w + 3\times 3\gamma_w = P_{max}$$

$$P_{max} = 13\times 9{,}810$$

$$= 127{,}530\ N/m^2$$

$$\boxed{P_{max} = 127{,}5\ kPa}$$

max. pressure will be at the bottom of the liquid that has a specific gravity of s=3

$$F_{CD} = PA$$

$$= (127530 - 1\times 3\times 9810)\times 1\,m^2$$

$$\boxed{F_{CD} = 98{,}1\ kN}$$

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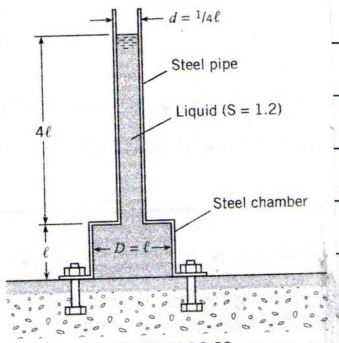
**3.22** The steel pipe and steel chamber shown in the figure together weigh 600 lbf. What force will have to be exerted on the chamber by all the bolts to hold it in place? The dimension $\ell$ is equal to 2.5 ft. *Note:* There is no bottom on the chamber—only a flange bolted to the floor.



PROBLEM 3.22

weight of the steel = Force exerted by bot
+ weight of the liquid
+ pressure acting the bottom of the free body

$$F_B + W_{liquid} + W_s = P_2 A_2$$

$$\frac{P_1}{\gamma} + Z_1 = \frac{P_2}{\gamma_{liquid}} + Z_2$$

$$0 + 5L = \frac{P_2}{1,2\,\gamma_w} + 0 \Rightarrow P_2 = 1,2\,\gamma_w\,5L$$

$$= 1,2 * 62,4 * 2,5 * 5$$

$$= 936 \text{ psfg}$$

Area $\Rightarrow A_2 = \frac{\pi D^2}{4} = \frac{\pi L^2}{4} = \frac{\pi * 2,5^2}{4} = 4,909 \text{ ft}^2$

weight of liquid

$$W_{liquid} = \left(A_2 l + \frac{\pi d^2}{4} * 4L\right)\gamma_{liquid}$$

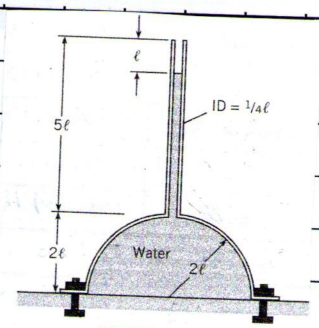
$$= \left(A_2 l + \frac{\pi L^3}{16}\right)(1,2)\,\gamma_w$$

$$= \left((4,909)(2,5) + \frac{\pi (2,5)^3}{16}\right) * (1,2) * (62,4)$$

$$= 1148,7 \text{ lbf}$$

$$F_B + (1148,7) + (600) = (936)(4,909) \Rightarrow \boxed{F_B = 2850 \text{ Lbf}}$$

3,23 What force must be exerted through the bolts to hold the dome in place? The metal dome and pipe weigh 6 kN. The dome has no bottom. Here $\ell = 80$ cm.




PROBLEM 3.23

$$\Sigma F_z = 0$$

$$P_{bottom} A_{bottom} + F_{bolts} - W_{H_2O} - W_{dome} = 0$$

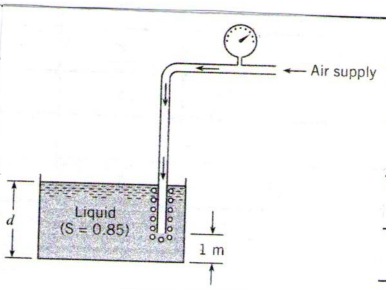
where

$$P_{bottom} A_{bottom} = 4{,}8 * 9{,}810 * \pi * 1{,}6^2 = 378{,}7 \text{ kN}$$

$$W_{H_2O} = 9810 \left(3{,}2 * \frac{\pi}{4} * 0{,}2^2 + \frac{2}{3} \pi * 1{,}6^3\right)$$
$$= 85{,}1 \text{ kN}$$

Then $F_{bolts} = -287{,}6$ kN

3.27 One means of determining the surface level of liquid in a tank is by discharging a small amount of air through a small tube, the end of which is submerged in the tank, and reading the pressure on the gage that is tapped into the tube. Then the level of the liquid surface in the tank can be calculated. If the pressure on the gage is ~~15~~ 20 kPa, what is the depth $d$ of liquid in the tank?



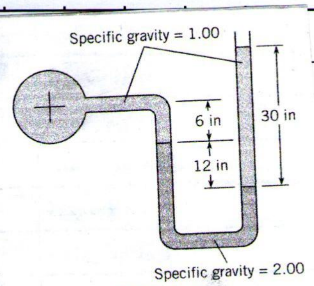

PROBLEM 3.27

$$P_{gage} - (d-1)\, \gamma_{liquid} = 0$$

$$20.000 - ((d-1) * 0{,}85 * 9810) = 0$$

$$d = 3{,}4 \text{ m}$$

**3.30** Is the gage pressure at the center of the pipe (a) negative, (b) zero, or (c) positive? Neglect surface tension effects and state your rationale.




PROBLEM 3.30

$$P_{pipe} + (0.5\,ft)(62.4\,lbf/ft^3) + (1\,ft)(2 \times 62.4\,lbf/ft^3) - (2.5\,ft)(62.4\,lbf/ft^3) = 0$$

$$P_{pipe} = (2.5 - 2 - 0.5)\,ft\,(62.4\,lbf/ft^3) = 0$$

$$P_{(center\ pipe)} = 0\ lbf/ft^3$$

**A differential manometer is shown in Fig. 2-33. Calculate the pressure difference between points *A* and *B*.**

$$p_A + [(0.92)(62.4)][(x+12)/12] - [(13.6)(62.4)](\tfrac{12}{12}) - [(0.92)(62.4)][(x+24)/12] = p_B$$

$$p_A - p_B = 906\ \text{lb/ft}^2$$

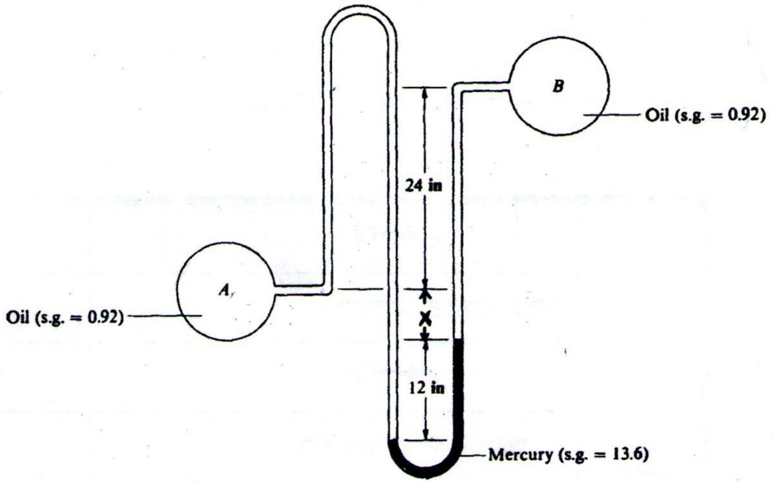



Fig. 2-33

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2.64 Determine the change in the elevation of the mercury in the left leg of the manometer of Fig. P2.64 as a result of an increase in pressure of 5 psi in pipe $A$ while the pressure in pipe $B$ remains constant.

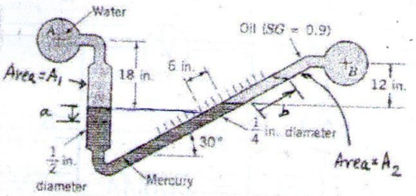


FIGURE P2.64

For the initial configuration:

$$p_A + \gamma_{H_2O}\left(\frac{18}{12}\right) - \gamma_{Hg}\left(\frac{6}{12}\sin 30°\right) - \gamma_{oil}\left(\frac{12}{12}\right) = p_B \qquad (1)$$

where all lengths are in ft. When $p_A$ increases to $p_A'$ the left column falls by the distance, $a$, and the right column moves up the distance, $b$, as shown in the figure. For the final configuration:

$$p_A' + \gamma_{H_2O}\left(\frac{18}{12}+a\right) - \gamma_{Hg}\left(a + \frac{6}{12}\sin 30° + b\sin 30°\right) - \gamma_{oil}\left(\frac{12}{12} - b\sin 30°\right) = p_B \qquad (2)$$

Subtract Eq. (1) from Eq. (2) to obtain

$$p_A' - p_A + \gamma_{H_2O}(a) - \gamma_{Hg}(a + b\sin 30°) + \gamma_{oil}(b\sin 30°) = 0 \qquad (3)$$

Since the volume of liquid must be constant $A_1 a = A_2 b$,

or

$$\left(\frac{1}{2}\text{ in.}\right)^2 a = \left(\frac{1}{4}\text{ in.}\right)^2 b$$

So that

$$b = 4a$$

Thus, Eq. (3) can be written as

$$p_A' - p_A + \gamma_{H_2O}(a) - \gamma_{Hg}(a + 4a\sin 30°) + \gamma_{oil}(4a\sin 30°) = 0$$

and

$$a = \frac{-(p_A' - p_A)}{\gamma_{H_2O} - \gamma_{Hg}(3) + \gamma_{oil}(2)} = \frac{-\left(5\ \frac{lb}{in.^2}\right)\left(144\ \frac{in.^2}{ft^2}\right)}{62.4\ \frac{lb}{ft^3} - \left(847\ \frac{lb}{ft^3}\right)(3) + (0.9)\left(62.4\ \frac{lb}{ft^3}\right)(2)}$$

$$= \underline{0.304\text{ ft (down)}}$$

**2.62** Determine the new differential reading along the inclined leg of the mercury manometer of Fig. P2.62, if the pressure in pipe $A$ is decreased 10 kPa and the pressure in pipe $B$ remains unchanged. The fluid in $A$ has a specific gravity of 0.9 and the fluid in $B$ is water.

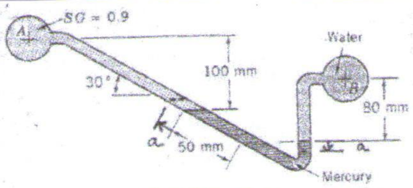


FIGURE P2.62

For the initial configuration:

$$p_A + \gamma_A (0.1) + \gamma_{Hg} (0.05 \sin 30°) - \gamma_{H_2O} (0.08) = p_B \qquad (1)$$

where all lengths are in m. When $p_A$ decreases left column moves up a distance, $a$, and right column moves down a distance, $a$, as shown in figure. For the final configuration:

$$p_A' + \gamma_A (0.1 - a \sin 30°) + \gamma_{Hg} (a \sin 30° + 0.05 \sin 30° + a) - \gamma_{H_2O} (0.08 + a) = p_B \qquad (2)$$

where $p_A'$ is the new pressure in pipe A.
Subtract Eq.(2) from Eq.(1) to obtain

$$p_A - p_A' + \gamma_A (a \sin 30°) - \gamma_{Hg}\, a (\sin 30° + 1) + \gamma_{H_2O} (a) = 0$$

Thus,

$$a = \frac{-(p_A - p_A')}{\gamma_A \sin 30° - \gamma_{Hg} (\sin 30° + 1) + \gamma_{H_2O}}$$

For $p_A - p_A' = 10$ kPa

$$a = \frac{-10 \frac{kN}{m^2}}{(0.9)(9.81 \frac{kN}{m^3})(0.5) - (133 \frac{kN}{m^3})(0.5+1) + 9.80 \frac{kN}{m^3}}$$

$$= 0.0540 \text{ m}$$

New differential reading, $\Delta h$, measured along inclined tube is equal to

$$\Delta h = \frac{a}{\sin 30°} + 0.05 + a$$

$$= \frac{0.0540 \text{ m}}{0.5} + 0.05 \text{ m} + 0.0540 \text{ m} = \underline{0.212 \text{ m}}$$

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**2.62** Determine the new differential reading along the inclined leg of the mercury manometer of Fig. P2.62, if the pressure in pipe $A$ is decreased 10 kPa and the pressure in pipe $B$ remains unchanged. The fluid in $A$ has a specific gravity of 0.9 and the fluid in $B$ is water.

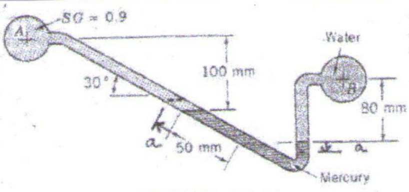



**FIGURE P2.62**

For the initial configuration:

$$p_A + \gamma_A(0.1) + \gamma_{Hg}(0.05 \sin 30°) - \gamma_{H_2O}(0.08) = p_B \quad (1)$$

where all lengths are in m. When $p_A$ decreases left column moves up a distance, $a$, and right column moves down a distance, $a$, as shown in figure. For the final configuration:

$$p_A' + \gamma_A(0.1 - a \sin 30°) + \gamma_{Hg}(a \sin 30° + 0.05 \sin 30° + a) - \gamma_{H_2O}(0.08 + a) = p_B \quad (2)$$

where $p_A'$ is the new pressure in pipe A.
Subtract Eq.(2) from Eq.(1) to obtain

$$p_A - p_A' + \gamma_A(a \sin 30°) - \gamma_{Hg}\, a(\sin 30° + 1) + \gamma_{H_2O}(a) = 0$$

Thus,

$$a = \frac{-(p_A - p_A')}{\gamma_A \sin 30° - \gamma_{Hg}(\sin 30° + 1) + \gamma_{H_2O}}$$

For $p_A - p_A' = 10$ kPa

$$a = \frac{-10 \frac{kN}{m^2}}{(0.9)(9.81 \frac{kN}{m^3})(0.5) - (133 \frac{kN}{m^3})(0.5+1) + 9.80 \frac{kN}{m^3}}$$

$$= 0.0540 \text{ m}$$

New differential reading, $\Delta h$, measured along inclined tube is equal to

$$\Delta h = \frac{a}{\sin 30°} + 0.05 + a$$

$$= \frac{0.0540 \text{ m}}{0.5} + 0.05 \text{ m} + 0.0540 \text{ m} = \underline{\underline{0.212 \text{ m}}}$$

### EXAMPLE 3.2 WATER PRESSURE IN A TANK

What is the water pressure at a depth of 35 ft in the tank shown?

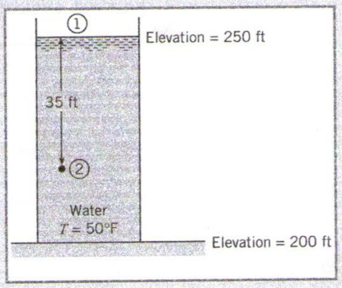
*Problem Definition*

**Situation:** Water is contained in a tank that is 50 ft deep.

**Find:** Water pressure (psig) at a depth of 35 ft.

**Properties:** Water (50°F), Table A.5: $\gamma = 62.4\ \text{lbf/ft}^3$.

**Sketch:**




*Plan*

Use the idea that piezometric head is constant. The steps are

1. Equate piezometric head at elevation 1 with piezometric head at elevation 2 (i.e., apply Eq. 3.7a).
2. Analyze each term in Eq. (3.7a).
3. Solve for the pressure at elevation 2.

*Solution*

1. Eq. (3.7a):

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

2. Term-by-term analysis of Eq. (3.7a) yields:
   - $p_1 = p_{atm} = 0$ psig
   - $z_1 = 250$ ft
   - $z_2 = 215$ ft
3. Combine steps 1 and 2:

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2$$

$$0 + 250\ \text{ft} = \frac{p_2}{62.4\ \text{lbf/ft}^3} + 215\ \text{ft}$$

$$p_2 = 2180\ \text{psfg} = \boxed{15.2\ \text{psig}}$$

*Review*

Remember! Gage pressure at the free surface of a liquid exposed to the atmosphere is zero.

### EXAMPLE 3.3 PRESSURE IN TANK WITH TWO FLUIDS

Oil with a specific gravity of 0.80 forms a layer 0.90 m deep in an open tank that is otherwise filled with water. The total depth of water and oil is 3 m. What is the gage pressure at the bottom of the tank?

*Problem Definition*

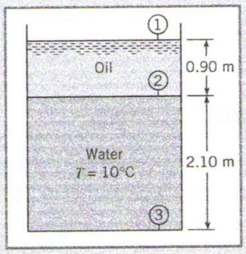
**Situation:** Oil and water are contained in a tank.

**Find:** Pressure (kPa gage) at the bottom of the tank.

**Properties:**

1. Oil (10°C), $S = 0.8$.
2. Water (10°C), Table A.5: $\gamma = 9810\ \text{N/m}^3$.

**Sketch:**




*Plan*

Use the idea that piezometric head is constant in a body of fluid with constant density. Recognize that pressure across the interface at elevation 2 is constant. The steps are

1. Find $p_2$ by applying the hydrostatic equation given in Eq. (3.7a).
2. Equate pressures across the oil-water interface.
3. Find $p_3$ by applying the hydrostatic equation given in Eq. (3.7a).

*Solution*

1. Hydrostatic equation (oil)

$$\frac{p_1}{\gamma_{oil}} + z_1 = \frac{p_2}{\gamma_{oil}} + z_2$$

$$\frac{0\ \text{Pa}}{\gamma_{oil}} + 3\ \text{m} = \frac{p_2}{0.8 \times 9810\ \text{N/m}^3} + 2.1\ \text{m}$$

$$p_2 = 7.063\ \text{kPa}$$

2. Oil-water interface

$$p_2|_{oil} = p_2|_{water} = 7.063\ \text{kPa}$$

3. Hydrostatic equation (water)

$$\frac{p_2}{\gamma_{water}} + z_2 = \frac{p_3}{\gamma_{water}} + z_3$$

$$\frac{7.063 \times 10^3\ \text{Pa}}{9810\ \text{N/m}^3} + 2.1\ \text{m} = \frac{p_3}{9810\ \text{N/m}^3} + 0\ \text{m}$$

$$\boxed{p_3 = 27.7\ \text{kPa gage}}$$

*Review*

Validation: Since oil is less dense than water, the answer should be slightly smaller than the pressure corresponding to a water column of 3 m. From Table F.1, a water column of $10\ \text{m} \approx 1$ atm. Thus, a 3 m water column should produce a pressure of about 0.3 atm = 30 kPa. The calculated value appears correct.

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**EXAMPLE 3.1 LOAD LIFTED BY A HYDRAULIC JACK**

A hydraulic jack has the dimensions shown. If one exerts a force $F$ of 100 N on the handle of the jack, what load, $F_2$, can the jack support? Neglect lifter weight.

***Problem Definition***

**Situation:** A force of $F = 100$ N is applied to the handle of a jack.

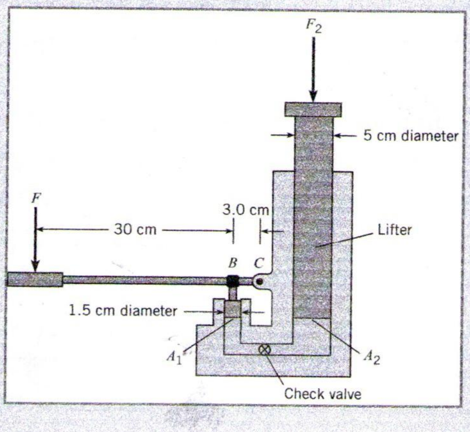
**Find:** Load $F_2$ in kN that the jack can lift.

**Assumptions:** Weight of the lifter component (see sketch) is negligible.

***Plan***

1. Calculate force acting on the small piston by applying moment equilibrium.
2. Calculate pressure $p_1$ in the hydraulic fluid by applying force equilibrium.
3. Calculate the load $F_2$ by applying force equilibrium.

**Sketch:**




***Solution***

1. Moment equilibrium

$$\sum M_C = 0$$

$$(0.33\text{ m}) \times (100\text{ N}) - (0.03\text{ m})F_1 = 0$$

$$F_1 = \frac{0.33\text{ m} \times 100\text{ N}}{0.03\text{ m}} = 1100\text{ N}$$

2. Force equilibrium (small piston)

$$\sum F_{\text{small piston}} = p_1 A_1 - F_1 = 0$$

$$p_1 A_1 = F_1 = 1100\text{ N}$$

Thus

$$p_1 = \frac{F_1}{A_1} = \frac{1100\text{ N}}{\pi d^2/4} = 6.22 \times 10^6\text{ N/m}^2$$

3. Force equilibrium (lifter)
   - Note that $p_1 = p_2$ because they are at the same elevation (this fact will be established in the next section).
   - Apply force equilibrium:

$$\sum F_{\text{lifter}} = F_2 - p_1 A_2 = 0$$

$$F_2 = p_1 A_2 = \left(6.22 \times 10^6\ \frac{\text{N}}{\text{m}^2}\right)\left(\frac{\pi}{4} \times (0.05\text{ m})^2\right) = \boxed{12.2\text{ kN}}$$

***Review***

The jack in this example, which combines a lever and a hydraulic machine, provides an output force of 12,200 N from an input force of 100 N. Thus, this jack provides a mechanical advantage of 122 to 1!

$$F = F_2 \frac{(L_1 + L_2)}{L_2} \quad \frac{A_2}{A_1}$$

N O T E B O O K

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**EXAMPLE 3.6 PRESSURE MEASUREMENT (U-TUBE MANOMETER)**

Water at 10°C is the fluid in the pipe of Fig. 3.11, and mercury is the manometer fluid. If the deflection $\Delta h$ is 60 cm and $\ell$ is 180 cm, what is the gage pressure at the center of the pipe?

*Problem Definition*

**Situation:** Pressure in a pipe is being measured using a U-tube manometer.

**Find:** Gage pressure (kPa) in the center of the pipe.

**Properties:**

1. Water (10°C), Table A.5, $\gamma = 9810\ \text{N/m}^3$.
2. Mercury, Table A.4: $\gamma = 133,000\ \text{N/m}^3$.

*Plan*

Start at point 1 and work to point 4 using ideas from Eq. (3.7c). When fluid depth increases, add a pressure change. When fluid depth decreases, subtract a pressure change.

*Solution*

1. Calculate the pressure at point 2 using the hydrostatic equation (3.7c).

$$p_2 = p_1 + \text{pressure increase between 1 and 2} = 0 + \gamma_m \Delta h_{12}$$
$$= \gamma_m (0.6\ \text{m}) = (133,000\ \text{N/m}^3)(0.6\ \text{m})$$
$$= 79.8\ \text{kPa}$$

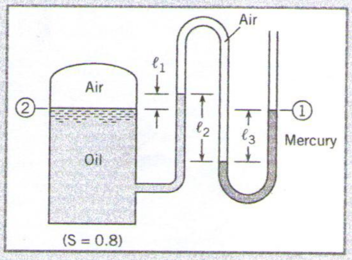
2. Find the pressure at point 3.
   - The hydrostatic equation with $z_3 = z_2$ gives
   $$p_3\big|_{\text{water}} = p_2\big|_{\text{water}} = 79.8\ \text{kPa}$$
   - When a fluid-fluid interface is flat, pressure is constant across the interface. Thus, at the oil-water interface
   $$p_3\big|_{\text{mercury}} = p_3\big|_{\text{water}} = 79.8\ \text{kPa}$$

3. Find the pressure at point 4 using the hydrostatic equation given in Eq. (3.7c).

$$p_4 = p_3 - \text{pressure decrease between 3 and 4} = p_3 - \gamma_w \ell$$
$$= 79{,}800\ \text{Pa} - (9810\ \text{N/m}^3)(1.8\ \text{m})$$
$$= 62.1\ \text{kPa gage}$$

**EXAMPLE 3.7 MANOMETER ANALYSIS**

**Sketch:** What is the pressure of the air in the tank if $\ell_1 = 40$ cm, $\ell_2 = 100$ cm, and $\ell_3 = 80$ cm?




*Problem Definition*

**Situation:** A tank is pressurized with air.

**Find:** Pressure (kPa gage) in the air.

**Assumptions:** Neglect the pressure change in the air column.

**Properties:** $\gamma_{Air} = 0$

1. Oil:

$$\gamma_{\text{oil}} = S\gamma_{\text{water}} = 0.8 \times 9810\ \text{N/m}^3 = 7850\ \text{N/m}^3.$$

2. Mercury, Table A.4: $\gamma = 133,000\ \text{N/m}^3$.

*Plan*

Apply the manometer equation (3.18) from elevation 1 to elevation 2.

*Solution*

Manometer equation

$$p_1 + \sum_{\text{down}} \gamma_i h_i - \sum_{\text{up}} \gamma_i h_i = p_2$$

$$p_1 + \gamma_{\text{mercury}} \ell_3 - \gamma_{\text{air}} \ell_2 + \gamma_{\text{oil}} \ell_1 = p_2$$

$$0 + (133,000\ \text{N/m}^3)(0.8\ \text{m}) - 0 + (7850\ \text{N/m}^3)(0.4\ \text{m}) = p_2$$

$$\boxed{p_2 = p_{\text{air}} = 110\ \text{kPa gage}}$$

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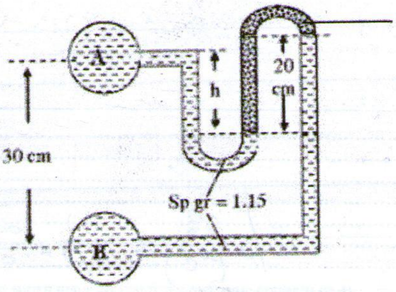
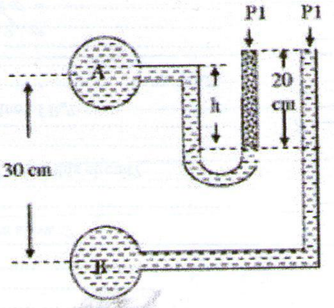
. **Compute the pressure difference between $A$ & $B$ in the figure.**




Guidance: The tubing system can be divided in two parts as shown below




$$P_A + h \times 1.15 \times 10^3 \times g$$
$$= P_1 + .20 \times 0.92 \times 10^3 \times g$$

$$P_1 + (.50 - h) \times 1.15 \times 10^3 \times g = P_B$$

Now $P_1 = P_1$

$$P_A + h \times 1.15 \times 10^3 \times g - 0.2 \times 0.92 \times 10^3 \times g$$
$$= P_B - (.5 - h) \times 1.15 \times 10^3 \times g$$

$$P_A - P_B = 0.2 \times 0.92 \times 10^3 \times g - 0.5 \times 1.15 \times 10^3 \times g$$
$$= 1.8 \times 10^3 - 5.64 \times 10^3$$
$$= -3.84 \times 10^3 \text{ kPa}$$
$$= -\frac{3.84 \times 10^3}{1 \times 10^3 \times 9.81}$$
$$= -391 \text{ mm of water}$$

Subject Date No.

**Example 2.9.** *A cylindrical tank of cross-sectional area 600 mm² and 2.6 m height is filled with water upto a height of 1.5 m and remaining with oil of specific gravity 0.78. The vessel is open to atmospheric pressure. Calculate:*

(*i*) *Intensity of pressure at the interface.*

(*ii*) *Absolute and gauge pressures on the base of the tank in terms of water head, oil head and N/m².*

(*iii*) *The net force experienced by the base of the tank.*

*Assume atmospheric pressure as 1.0132 bar.*

**Solution.** *Given:* Area of cross-section of the tank, $A = 600\ mm^2 = 600 \times 10^{-6}$;

sp.gr. of oil = 0.78; $p_{atm.} = 1.0132$ bar.

(*i*) **Intensity of pressure at the interface:**

The pressure intensity at the interface between the oil and water is due to 1.1 m of oil and is given by,

$$p_{interface} = wh$$
$$= (0.78 \times 9810) \times 1.1$$
$$= \mathbf{8417\ N/m^2\ (Ans.)}$$

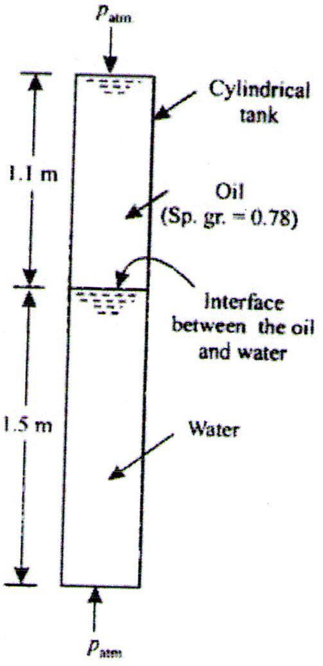



**Fig. 2.7**

(*ii*) **Absolute and gauge pressure on the base of the tank:**

Pressure at the base of the tank

= Pressure at the interface (due to 1.1 m of oil) + pressure due to 1.5 m of water

*i.e.,* $p_{base\,(gauge)} = 8417 + (9810 \times 1.5)$

$$= \mathbf{23132\ N/m^2\ (gauge)\ (Ans.)}$$

$$= \frac{23132}{9810} = \mathbf{2.358\ m\ of\ water\ (gauge)\ (Ans.)}$$

$$= \frac{23132}{0.78 \times 9810} = \mathbf{3.023\ m\ of\ oil\ (gauge)\ (Ans.)}$$

Atmospheric pressure, $p_{atm.} = 1.0132$ bar

$= 1.0132 \times 10^5\ N/m^2$

$$= \frac{1.0132 \times 10^5}{9810} = 10.328\ \text{m of water}$$

$$= \frac{1.0132 \times 10^5}{0.78 \times 9810} = 13.241\ \text{m of oil}$$

Absolute pressure = Atmospheric pressure + gauge pressure

$$p_{base}\text{(absolute)} = 10.328 + 2.358 = \mathbf{12.686\ m\ of\ water\ (Ans.)}$$
$$= 13.241 + 3.023 = \mathbf{16.264\ m\ of\ oil\ (Ans.)}$$
$$= 101320 + 23132 = \mathbf{124452\ N/m^2\ (Ans.)}$$

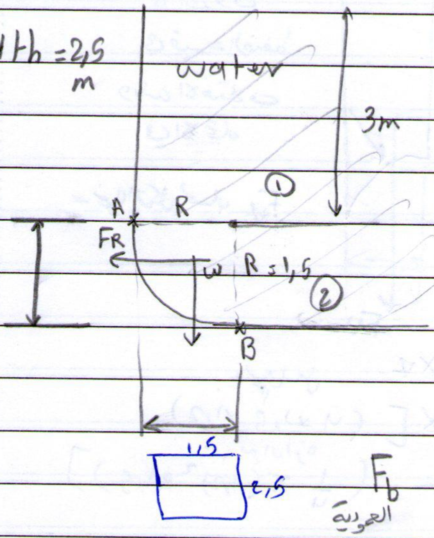
(*iii*) **The net force experienced by the base of the tank:**

$$F\ (= P) = p_{base}(\text{gauge}) \times \text{cross-sectional area}$$
$$= 23132 \times 600 \times 10^{-6} = \mathbf{13.879\ N\ (Ans.)}$$

Poletechnic Lecture Note

Subject Date No.

## * Force on curved surface :-



$F_R$ : hydrostatic force

width = 2,5 m

$w = \rho g V = \gamma V$

$= \gamma ( [2,5 * 3 * 1,5] + [\frac{1}{4} (\pi (1,5)^2 (2,5)] )$

$= \boxed{153,3 kN}$

$V_2$ ربع دائرة

$\frac{1}{4}$ * مساحة الدائرة

$F_h = \gamma \bar{y} A \sin\alpha$ (الحوية)

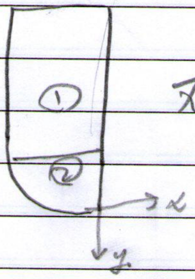
$= 9810 * 3,75 * (1,5 * 2,5) * \sin 90 = \boxed{138 kN}$

sin 90

$y_{cp} = \bar{y} + \frac{I}{\bar{y} A}$

$= 3,75 + \frac{\frac{1}{12} [2,5 * 1,5^3]}{3,75 [1,5 * 2,5]} = 3,8 m$

$\bar{y} = 3 + \frac{R}{2} = 3,75$ m



$\bar{x} = \frac{\Sigma x_i A_i}{\Sigma A_i} = \frac{0,75 [1,5 * 3] + \frac{4}{3\pi} (1,5) [\frac{1}{4} \pi R^2]}{(1,5 * 3) (\frac{1}{4} \pi (1,5)^2]}$

$\bar{x} = 0,718 m$

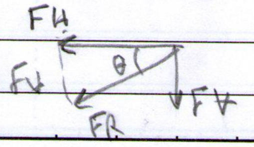
$A_{projection}$

1,5

2,5

$I = \frac{1}{12} b h^3$

$= \frac{1}{12} [2,5 * 1 ...$



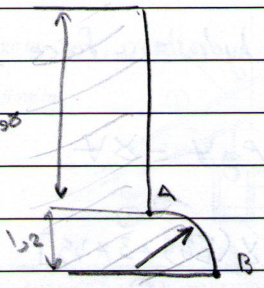
$F_R = \sqrt{F_V^2 + F_H^2} = \sqrt{153,7^2 + (138)^2} = 206,5 kN$

$\theta = \tan^{-1} \frac{F_V}{F_H} = 48,1$

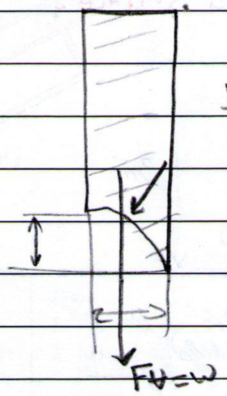
NOTEBOOK

Subject Date No.

*ex:-




$w = 1,6m$




لا اختلاف
في قيمة الضغط
ولكن الاختلاف
في الاتجاه

* نحول الشكل لحساب $F_V$

$F_V = W = \gamma V$

الحجم الكلي

$= \gamma [ (4 * 1,5 * 1,2) -$

حجم ربع الدائرة

$( \frac{1}{4} \pi (1,2)^2 * 1,5 ) ]$

$= 53989 \; N$

$F_H = \gamma A \bar{y} \sin\alpha$

$= 9810 [1,2 * 1,5] [3,4] \sin 90$

$= 66037 \; N$

$\bar{y} = 2,8 + \frac{1,2}{2}$

$= 3,4 m$

$F_R = \sqrt{F_V^2 + F_H^2} = 80741 N$

Subject problem :- Date No.

1.3 A liquid at 20°C has a relative density of 0.80 and a kinematic viscosity of 2.3 centistoke. Determine its (i) unit weight and (ii) dynamic viscosity in Pa.s.

*Solution:*

(i) Unit weight $\gamma = \rho g = (\rho_{water} \times RD) \times g$

Taking $\rho_{water}$ at 20° as 998 kg/m$^3$,

$$\gamma = 998 \times 0.8 \times 9.81$$
$$= 7832.3 \text{ N/m}^3 = \mathbf{7.832 \text{ kN/m}^3}$$

(ii) Dynamic viscosity $\mu = \nu\rho$

$\nu = 2.3$ centistoke $= 2.3 \times 10^{-6}$ m$^2$/s

$\rho = 998 \times 0.8 = 798.4$ kg/m$^3$

$\mu = 2.3 \times 10^{-6} \times 798.4 = \mathbf{1.836 \times 10^{-3} \text{ Pa.s.}}$

1.4 The space between two parallel plates kept 3 mm apart is filled with an oil of dynamic viscosity 0.2 Pa.s. What is the shear stress on the lower fixed plate, if the upper one is moved with a velocity of 1.50 m/s? (Refer Fig 1.2).

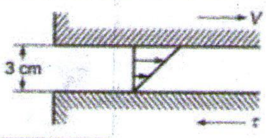



**Fig. 1.2**

*Solution:* Since the gap between the plates is very small, a linear variation of velocity can be assumed.

$$\frac{du}{dy} = \frac{V}{h} = \frac{1.50}{3 \times 10^{-3}} = 500(s^{-1})$$

$\tau$ = Shear stress on the bottom plate

$$= \mu \frac{du}{dy} = 0.2 \times 500 = 100 \text{ N/m}^2$$

1.5 The velocity distribution in a viscous flow over a plate is given by

$$u = 4y - y^2 \text{ for } y \le 2 \text{ m}$$

where $u$ = velocity in m/s at a point distant $y$ from the plate. If the coefficient of dynamic viscosity is 1.5 Pa.s determine the shear stress at $y = 0$ and at $y = 2.0$ m.

*Solution:* Given

$$u = 4y - y^2$$

Therefore $\frac{du}{dy} = 4 - 2y$

Shear stress $\tau = \frac{du}{dy} = \mu(4 - 2y)$

At $y = 0$, $\tau_0 = 4\mu = 4 \times 1.5 = \mathbf{6.0 \text{ Pa.s}}$

At $y = 2.0$ m, $\tau_2 = \mu(4 - 4) = \mathbf{0}$

**1.6** The velocity distribution near the solid wall at a section in a laminar flow is given by

$$u = 5.0 \sin(5\pi y)$$

for $y \leq 0.10$ m. Compute the shear stress at a section at (i) $y = 0$, (ii) $y = 0.05$ m and (iii) $y = 0.10$ m. The dynamic viscosity of the fluid is 5 poise.

*Solution:* Given,

$$\mu = 5 \text{ poise} = \frac{5}{10} \text{ Pa.s}$$

Since $u = 5.0 \sin(5\pi y)$

$$\frac{du}{dy} = 5.0 \times 5\pi \cos(5\pi y)$$

Shear stress $\tau = \mu \frac{du}{dy} = \frac{5}{10} \times 25\pi \cos(5\pi y) = 12.5\pi \cos(5\pi y)$

(i) At $y = 0$, $\tau = 12.5\pi \cos(0) = 12.5\pi =$ **39.27 N/m²**

(ii) At $y = 0.05$ m, $\tau = 12.5\pi \cos(5\pi \times 0.05)$ $= 12.5\pi \times 0.707 =$ **27.76 N/m²**

(iii) At $y = 0.10$ m, $\tau = 12.5\pi \cos(5\pi \times 0.1) =$ **0**

**1.7** A 90 N rectangular solid block slides down a 30° inclined plane. The plane is lubricated by a 3 mm thick film of oil of relative density 0.90 and viscosity 8.0 poise. If the contact area is 0.3 m², estimate the terminal velocity of the block (Refer. Fig. 1.3).

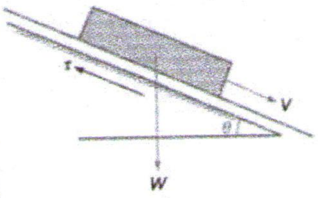



**Fig. 1.3**

*Solution:* Given

$$W = 90 \text{ N}$$
$$V = \text{terminal velocity}$$
$$\theta = 30°$$

At the terminal velocity, the sum of the forces acting on the block in the direction of its motion is zero. Hence

$$W \sin\theta - \tau A = 0$$

where $\tau$ = shear stress on the block and
$A$ = area of the block.

$$\tau = \mu \frac{du}{dy} = \mu \frac{V}{h} \text{ where } h = \text{thickness of oil film}$$

$$\mu = 8 \text{ poise} = \frac{8}{10} \text{ Pa.s} = 0.8 \text{ Pa.s}$$

$$h = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$$

$$A = 0.3 \text{ m}^2$$

Substituting the various values in the above equation,

$$90 \sin 30° - \frac{(0.8\,V)}{3 \times 10^{-3}} \times (0.3) = 0$$

$$\therefore \quad V = \frac{45}{80} = \mathbf{0.5625 \text{ m/s}}$$

Subject Date No.

**1.9** A cylindrical shaft of 90 mm diameter rotates about a vertical axis inside a fixed cylindrical tube of length 50 cm and 95 mm internal diameter. If the space between the tube and the shaft is filled by a lubricant of dynamic viscosity 2.0 poise, determine the power required to overcome viscous resistance when the shaft is rotated at a speed of 240 rpm (Refer. Fig. 1.5).

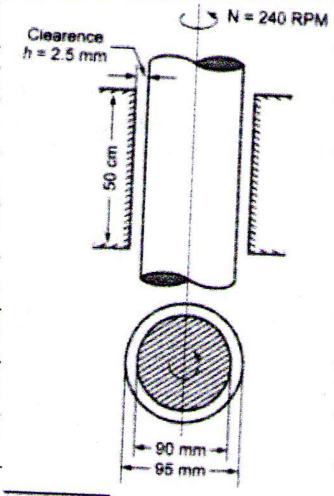



**Fig. 1.5**

*Solution:*

$V$ = Circumferential velocity of the shaft

$$= \omega r = \frac{2\pi N}{60} \times r$$

$$= \frac{2\pi \times 240}{60} \times \left(\frac{0.090}{2}\right) = 1.131 \text{ m/s}$$

Clearance $h = \frac{95 - 90}{2} = 2.5$ mm

Assuming linear variation of velocity across the gap,

Velocity gradient $\frac{du}{dr} = \frac{V}{h}$

$$= \frac{1.131}{2.5 \times 10^{-3}} = 452.4 \text{ s}^{-1}$$

$$\mu = 2.0 \text{ poise} = 0.2 \text{ Pa.s}$$

Shear stress on the shaft $\tau = \mu \frac{du}{dr}$

$$= 0.2 \times 452.4 = 90.48 \text{ Pa}$$

Shear force $F_s = \tau \times 2\pi r \times L$

$$= 90.48 \times \left(2\pi \times \frac{0.09}{2}\right) \times 0.50$$

$$= 12.791 \text{ N}$$

Torque $T = F_s r$

$$= 12.791 \times \frac{0.09}{2} = 0.6756 \text{ N.m}$$

Power required $= P = \frac{2\pi N}{60} \cdot T$

$$= \frac{2\pi \times 240}{60} \times 0.6756 = \boxed{14.5 \text{ W}}$$

Poletechnic Lecture Note

Subject Date No.

**1.10** A sleeve 10 cm long encases a vertical metal rod 3.0 cm in diameter with a radial clearance of 0.02 mm. If when immersed in an oil of viscosity 6.0 poise, the effective weight of the sleeve is 7.5 N, will the sleeve slide down the rod and if so at what velocity? (Refer Fig 1.6)

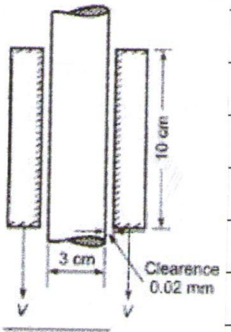



**Fig. 1.6**

*Solution:* Given

$$\mu = 6 \text{ poise} = 0.6 \text{ Pa.s}$$
$$h = 0.02 \text{ mm} = 0.02 \times 10^{-3} \text{ m}$$

Let $V$ = Velocity of the sleeve when sliding down.

$$\text{Shear stress } \tau = \mu \frac{du}{dr} = \mu \frac{V}{h}$$
$$= (0.6) \times \frac{V}{2\times10^{-5}} = 3 \times 10^4 \text{ V}$$

At terminal velocity

Shear force = Submerged weight of the sleeve.

$$\therefore \quad (2\pi rL) \times \tau = W_s$$
$$\left(2\pi \times \frac{0.03}{2} \times \frac{10}{100} \times 3\times10^4\right)V = 7.5$$
$$282.74\ V = 7.5$$
$$V = 0.02653 \text{ m/s} = \mathbf{2.66 \text{ cm/s}}$$

**1.11** A hydraulic lift used for lifting automobiles has a 25 cm diameter ram which slides in a 25.018 cm diameter cylinder, the annular space being filled with oil having a kinematic viscosity of 3.7 $cm^2$/s and relative density of 0.85. If the rate of travel of the ram is 15 cm/s find the frictional resistance when 3.3 m of ram is engaged in the cylinder.

*Solution:*

$$\mu = \nu\rho = 3.7 \times 10^{-4} \times 0.85 \times 998$$
$$= 0.3139 \text{ Pa.s}$$
$$\text{Shear stress } \tau = \mu \frac{du}{dy} = \mu \frac{V}{h}$$
$$= 0.3139 \times \frac{0.15}{(25.018 - 25.00)/(2\times10^2)}$$
$$= 523.1 \text{ N/m}^2$$

Frictional resistance $F_s = A\tau$

$$F_s = (\pi DL) \times \tau$$
$$= \pi \times \frac{25}{100} \times 3.3 \times 523.1$$
$$= 1356 \text{ N} = \mathbf{1.356 \text{ kN}}$$

**1.12** A circular disc of radius $R$ is kept at a small height above a fixed bed by means of a layer of oil of viscosity $\mu$. If the disc is rotated at an angular velocity $\omega$, obtain an expression for viscous torque on the disc (Refer Fig 1.7).

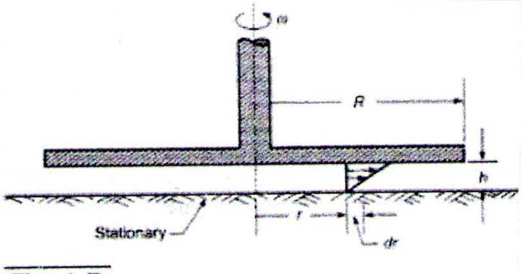



**Fig. 1.7**

*Solution:* Consider an element of disc of width $dr$ at a radial distance $r$.

Velocity at this radius $= V = r\omega$.

Assuming linear variation of velocity with depth in the gap $h$.

Shear stress $\tau = \mu \dfrac{V}{h} = \dfrac{\mu}{h} \omega r$

Viscous torque on the element

$$dT = \frac{\mu}{h} \omega r \,(2\pi r dr) r$$

$$= \frac{\mu\omega}{h} 2\pi r^3 dr$$

Total torque $\quad T = \displaystyle\int_0^R dT = \int_0^R \frac{\mu\omega}{h} 2\pi r^3 dr$

$$= \left[\frac{\mu\omega}{h} . 2\pi \frac{r^4}{4}\right]_0^R$$

or $\quad \mathbf{T = \dfrac{1}{2} \dfrac{\pi\mu\omega}{h} . R^4}$

**1.15** If the surface tension at air-water interface is 0.073 N/m, what is the pressure difference between inside and outside of an air bubble of diameter 0.01 mm?

*Solution:* An air bubble has only one surface. Hence

$$\Delta p = \frac{2\sigma}{R}$$

$$= \frac{2 \times 0.073}{\left(\frac{0.01}{2}\right) \times 10^{-3}} = 29200 \text{ N/m}^2$$

$$= \mathbf{29.2 \text{ kPa}}$$

**1.16** If the surface tension at the soap-air interface is 0.088 N/m, calculate the internal pressure in a soap bubble of 3 cm diameter.

*Solution:* In a soap bubble, there are two interfaces. Hence

$$\Delta p = \frac{4\sigma}{R} = \frac{4 \times 0.088}{\left(\frac{3}{2} \times 10^{-2}\right)}$$

$$= \mathbf{23.47 \text{ N/m}^2 \text{ above atmospheric pressure}}$$

**1.17** A U-tube has two limbs of internal diameter 6 mm and 16 mm respectively and contains some water. Calculate the difference in water levels in the two limbs. Surface tension of water = 0.073 N/m.

*Solution:* Let $h$ = difference in water levels in the two limbs. By assuming the angle of contact $\theta = 0°$,

$$\Delta p = \rho g h = \left(\frac{2\sigma}{R_1} - \frac{2\sigma}{R_2}\right)$$

$$h \times 998 \times 9.81 = 2 \times 0.073 \left(\frac{1}{3 \times 10^{-3}} - \frac{1}{8 \times 10^{-3}}\right)$$

$$h = 3.1 \times 10^{-3} \text{ m} = \mathbf{3.1 \text{ mm}}$$

**1.13** Two coaxial cylinders 10 cm and 9.75 cm in diameter and 2.5 cm high have both their ends open and have a viscous liquid filled in between. A torque of 1.2 N.m is produced on the inner cylinder when the outer one rotates at 90 rpm. Determine the coefficient of viscosity of the liquid (Refer Fig 1.8).

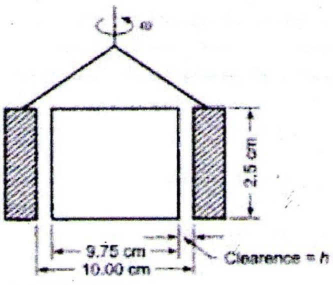



**Fig. 1.8**

*Solution:* Tangential velocity

$$V = \frac{\pi DN}{60} = \frac{\pi \times 0.10 \times 90}{60} = 0.4712 \text{ m/s}$$

Radial clearance $h = \frac{(10.00 - 9.75)}{2}$ cm $= 0.125$ cm or $0.00125$ m

Velocity gradient $\frac{du}{dr} = \frac{V}{h} = \frac{0.4712}{0.00125} = 377$

Shear stress $\tau = \mu \frac{du}{dr} = 377\mu$

Shear force $F_s = \tau \times \text{area}$

$$= 377\mu\left(2\pi \times \frac{0.1}{2} \times \frac{2.5}{100}\right)$$

$$= 2.961\mu$$

Torque $= F_s \times \text{lever arm} = F_s\, r$

$$T = 2.961\,\mu \times \frac{(0.1)}{2} = 0.14805\,\mu$$

But Torque $T = 1.2$ N.m

Therefore $0.14805\,\mu = 1.2$

$\therefore$ $\mu = \frac{120}{0.14805} = \mathbf{8.106\ Pa.s}$

**1.18** A clean tube of internal diameter 3 mm is immersed in a liquid with a coefficient of surface tension = 0.48 N/m. The angle of contact of the liquid with the glass can be assumed to be 130°. The density of the liquid is 13,600 kg/m³. What would be the level of the liquid in the tube relative to the free surface of the liquid outside the tube.

*Solution:* The liquid in the tube rises (or falls) due to capillarity. The capillary rise (or fall)

$$h = \frac{2\sigma\cos\theta}{\gamma R}$$

Here $R = \frac{3}{2}$ mm $= 1.5 \times 10^{-3}$ m

$\theta = 130°$, $\sigma = 0.48$ N/m

$\gamma = \rho g = 13.6 \times 10^3 \times 9.81$

$\therefore$ $h = \frac{2 \times 0.48 \times \cos 130°}{(13.6 \times 10^3 \times 9.81) \times (1.5 \times 10^{-3})}$

$= -3.08 \times 10^{-3}$ m $= \mathbf{-3.08\ mm}$

Therefore, there is a capillary depression of 3.08 mm.

Subject Date No.

**1.19** Assuming that the interstices in a clay are of size equal to one tenth the mean diameter of the grain, estimate the height to which water will rise in a clay soil of average grain diameter 0.05 mm. Assume surface tension at air-water interface as 0.073 N/m.

*Solution:*

Diameter of pores = $2R = 0.005$ mm

$$R = 0.00025 \text{ mm} = 2.5 \times 10^{-6} \text{ m}$$

$$\Delta h = \frac{2\sigma}{\gamma R} \text{ by assuming } \theta = 0^\circ$$

$$= \frac{2 \times 0.073}{(9.81 \times 998) \times (2.5 \times 10^{-6})}$$

$$= \mathbf{5.95 \text{ m}}$$

**1.27** Determine the pressure increase required to reduce the volume of water by 1.5%, if its bulk modulus of elasticity is $2.2 \times 10^9$ Pa.

*Solution:* Let $V$ = Volume of water

Change in volume $= dV = -\frac{1.5}{100} V = -0.015 V$

$$-\frac{dV}{V} = 0.015$$

Increase in pressure $= \Delta p = \left(-\frac{dV}{V}\right) K$

$$= 2.2 \times 10^9 \times 0.015$$

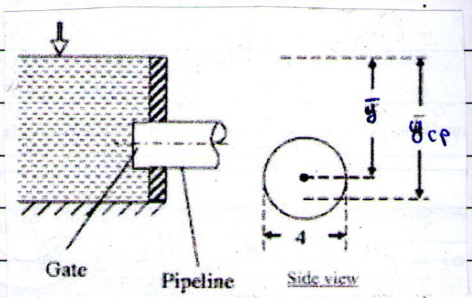
$$= 3.3 \times 10^7 \text{ Pa}$$

$$= \mathbf{3.3 \times 10^4 \text{ kPa}}$$

N O T E B O O K

Subject Date No.

**18. Flow in a pipeline of 4 m in diameter is controlled by a circular gate. The sp gravity of the fluid is 0.9. The pressure at the centre of gate is 180 kN/m². Find (1) total pressure force on the gate and (2) the depth of centre of pressure from the free surface.**




$F = \gamma_F A \bar{y} \sin\alpha$

$= [8829] * [12{,}56]$

$[20{,}387] * \sin 90$

$= 2261092\ N$

$\approx 2261\ kN$

$\gamma_F = \gamma_w * 0{,}9 = 9810 * 0{,}9$

$= 8829$

$\bar{y}_{cp} = \bar{y} + \frac{I}{\bar{y}A}$

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(4)^2 = 12{,}56$

$= 20{,}387 + \frac{12{,}56}{[20{,}387]*[12{,}56]}$

$P_c = \gamma \bar{y}$

$= \rho g \bar{y}$

$= 20{,}436\ m$

$180 \times 10^3 = \bar{y} * 0{,}9 * 9810$

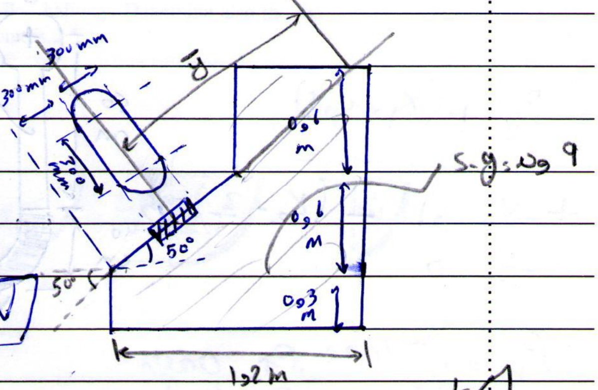
$\bar{y} = 20{,}387\ m$

$I = \frac{1}{64}\pi d^4$

$= \frac{\pi r^4}{4} = \boxed{12{,}56}$

Subject                                        Date                    No.

* determine the hydrostatic Force on the gate and the center of pressure?!



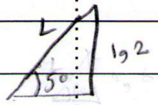

$F_R = \gamma \bar{y} A \sin\alpha$

$= [8829][1.116][0.1607][\sin 50] = \boxed{1212.957\ N}$

$y_{cp} = \bar{y} + \frac{I}{\bar{y}A}$

$= 1.116 + \frac{1.0726 \times 10^{-3}}{[1.116][0.1607]}$

$\boxed{y_{cp} = 1.122\ m}$




$L = \frac{1.2}{\sin 50} = 1.566$

$\alpha = 50$

$\bar{y} = 1.566 - 0.45 = 1.116$

$A = A_{semicercile} + A_{squre}$

$= 2 * \left[\frac{\pi r^2}{2}\right] + D^2$

$= \pi (0.15)^2 + 0.3^2$

$= 0.1607\ m^2$

$\gamma_F = s.g\ \gamma_w$

$= 0.9 \times 9810$

$= 8829$

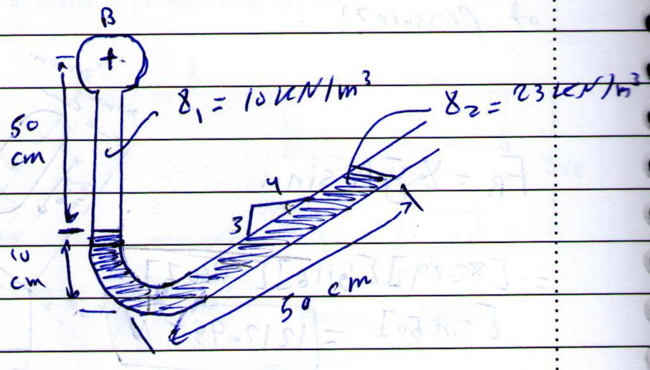
$I = \frac{\pi}{4} R^4 + \frac{1}{12} bh^3$

$= \frac{\pi}{4}(0.15)^4 + \frac{1}{12}(0.3)^4$

$= 1.0726 \times 10^{-3}$

Subject ✱ Date No.

* what is the pressure at the center of pipe B?




$P_B + (\gamma_1 * 0.5)$

$+ \gamma_2 (0.1) - (\gamma_2 * \frac{3}{5}) * 0.5$

$= 0$

$P_B = -1000$ Pa gage

Subject — Date — No.

ex:-

A concrete culvert that contains water is 2.0 m in diameter. Determine the forces exerted on the portion labeled *A–B* in Figure 2.18 if the culvert is filled halfway. Determine also the location of the forces. Culvert length (into the paper) from joint to joint is 2.5 m.

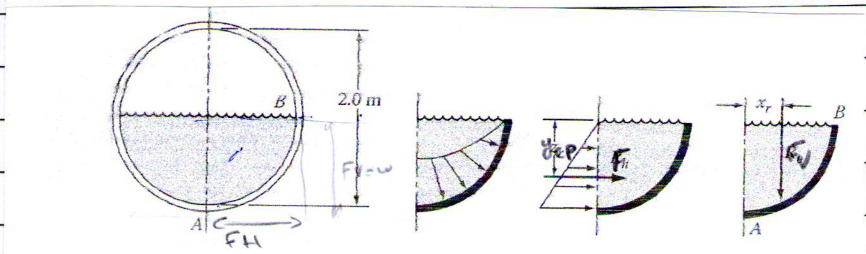



FIGURE 2.18 Cross section of a half-filled culvert.

sol :

① The horizontal force

$$F_H = \gamma A \bar{Z} \to \bar{y}$$
$$= [9810][2.5] * [0.5]$$
$$= 12.3\ kN$$

$A = 1 * 2.5 = 2.5$

$\bar{Z} = \frac{D}{4} = \frac{2}{4} = 0.5$

$$y_{cp} = \bar{y} + \frac{I}{\bar{y}A} = 0.5 + \frac{0.2083}{0.5 * 2.5}$$
$$= 0.667\ m$$

below the surface

$I_{xx} = \frac{bh^3}{12} = \frac{2.5 * 1^3}{12} = 0.2083\ m^4$

② $F_V$ = weight of the fluid above AB

$$F_V = W = \gamma V$$
$$= (9810)\left(\frac{\pi}{4}(1)^2(2.5)\right) = 19.25\ kN$$

$$x_{cp} = \frac{4R}{3\pi} = \frac{4(1)}{3\pi} = 0.42\ m$$ from the center line

Subject Date No.

ex:- When the culvert described in Example 2.9 was installed and still empty, it was buried halfway in mud (Figure 2.19). Determine the forces acting on half the submerged portion, assuming that the mud has a density equal to that of water.

$\bar{y} = 0.5$ , $A = 2.5$

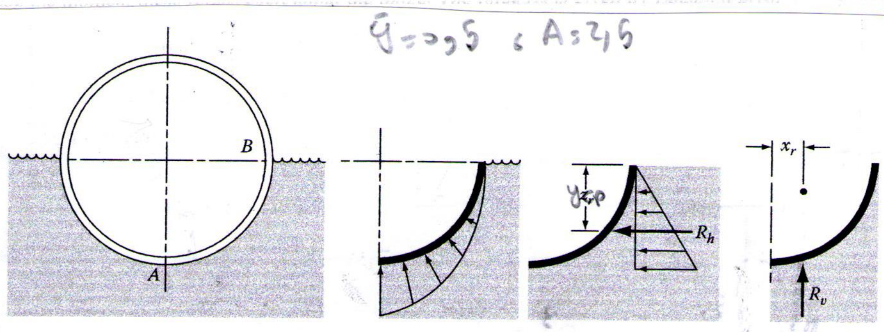



**FIGURE 2.19** A partially submerged culvert.

$$F_h = \gamma \bar{y} A$$
$$= [9810][0.5][2.5] = 12.3 kN$$

$$y_{cp} = \bar{y} + \frac{I}{\bar{y}A} = 0.5 + \frac{\frac{2.5 \times 1^3}{12}}{0.5 \times (2.5)} = 0.667 m$$

below the free surface

$$F_V = W = \gamma V$$
$$= (9810)(\pi(1)^2(2.5))$$
$$= 19.25 N$$

$$x_{cp} = \frac{4R}{3\pi} = \frac{(4\times1)}{(3)(\pi)} = 0.42$$ from the center line

Subject Date No.

Gate *AB* in Fig. 5-13*a* is a quarter circle 8 ft wide into the paper. Find the force *F* just sufficient to prevent rotation about hinge *B*. Neglect the weight of the gate.

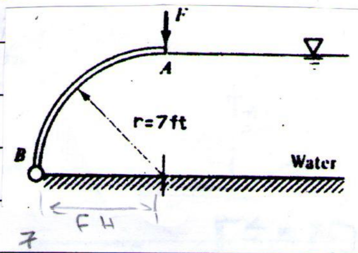
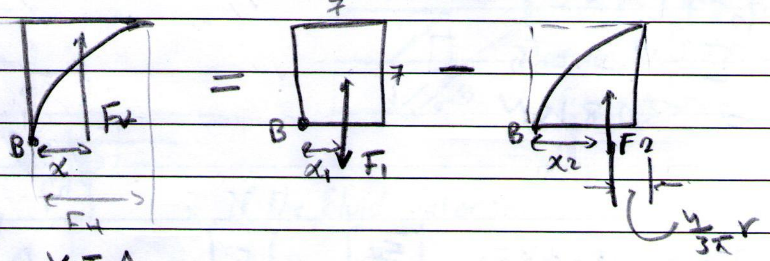



**Fig. 5-13(*a*)**



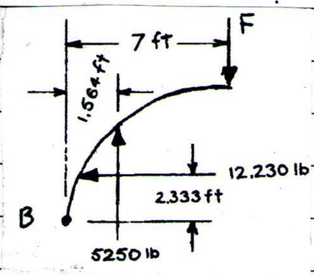

$$F_H = \gamma \bar{y} A$$
$$= 62.4 \left[\frac{7}{2}\right]\left[7\times 8\right] = 12230 \text{ lb}$$

$$F_V = F_1 - F_2$$
$$= 62.4\left[7\times 7\times 8\right] - 62.4\left[\frac{1}{2}\times\frac{\pi(7)^2}{2}\times 8\right]$$
$$= 24461 - 19211 = 5250 \text{ lb}$$

$\Sigma M_B = 0$

$$5250(x) = (24461)\left(\frac{7}{2}\right) - (19211)\left(\frac{4(7)}{3\pi}\right)$$

$$\boxed{x = 1.564 \text{ ft}}$$



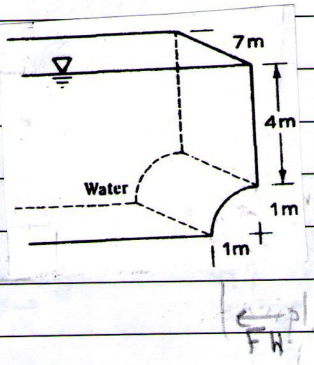

$\Sigma M_B = 0$

$$7F - (2.333)(12230) - (1.564)(5250) = 0$$

$$F = 5249 \text{ lb}$$

Subject Date No.

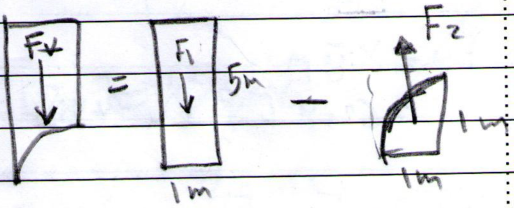
* compute the horizontal and vertical components of the hydrostatic force on the quarter-circle face on the tank shown in figure?




$F_H = \gamma A \bar{y}$

$= 9{,}79\,[1 * 7]\,[4 + 0{,}5]$

$= 308\ kN$




$F_V = F_1 - F_2$

$= [9{,}79\,[7 * 1 * 5]] - [9{,}79\,[\frac{\pi (1)^2}{4} * 7]]$

$= 289\ kN$

N O T E B O O K

2nd. exam

Subject Date No.

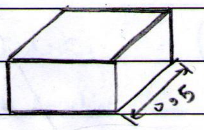
## * Buoyancy and stability :-

* Buoyancy :-

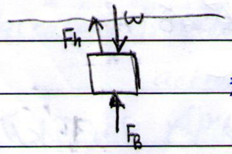
$F_B = \gamma_F \forall_d$ ⇒ للجسم المغمور

ex:- cube bronze :-

$\gamma = 86.9 \ kN/m^3$




Find :* مقدار القوة التي تؤثر على أن يكون الجسم مغمور بالكامل ؟




* if the Fluid water :-

$w = \gamma \forall = 86.9 * (0.5)^3 = 10.86 \ KN$

$F_B = \gamma_F \forall_d = 9.81 * (0.5)^3 = 1.23 \ KN$

$F_h + F_B - w = 0$

$F_h = w - F_B = 10.86 - 1.23$
$= 9.63 \ KN$

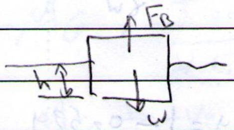
* if the fluid mercury (Hg):-

$F_B = \gamma_F \cdot \forall_d = 13.54 * (0.5)^3 * 9.81$
$= 16.6$

$F_h = w - F_B = 10.86 - 16.6 = -5.74$

مغمور جزئي

* if $F_h = 0$



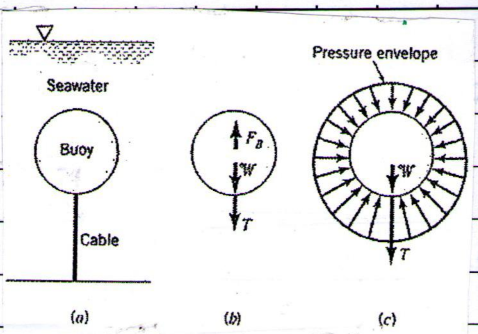

$w = F_B$

$10.86 = 13.54 * 9.81 * \forall_d \Rightarrow V_d = 0.0817 \ m^3$

$V_d = A * H \Rightarrow H = \frac{V_d}{A} = \frac{0.0817}{0.25} = 0.327 \ m$

Subject Date No.

A spherical buoy has a diameter of 1.5 m, weighs 8.50 kN, and is anchored to the seafloor with a cable as is shown in Fig. E2.10*a*. Although the buoy normally floats on the surface, at certain times the water depth increases so that the buoy is completely immersed as illustrated. For this condition what is the tension of the cable? $\gamma = 10.1\ kN/m^3$




$F_B - W - T = 0$

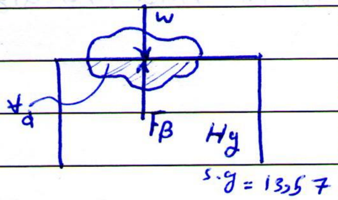
$\boxed{T = F_B - W}$

$F_B = \gamma \forall = (10.1)\left(\frac{\pi d^3}{6}\right)$

$F_B = (10.1 \times 10^3)\left(\frac{\pi}{6}(1.5\,m)^3\right) = 1.785 \times 10^4\,N$

$T = (1.785 \times 10^4) - (0.850 \times 10^4) = 9.35\,kN$

ex: what fraction of the volume of a solid piece of metal of density 7250 kg/m³ floats above the surface of a container of mercury of specific gravity 13.57??




$W = F_B$

$mg = \gamma \forall_d$

$\rho g \forall_{metal} = (\rho g)_{Hg} \forall_d$

$\frac{\forall_d}{\forall_m} = \frac{(\rho g)_m}{(\rho g)_{Hg}} = \frac{7250}{13570} = 0.534$

$s.g = \frac{\gamma_f}{\gamma_w}$

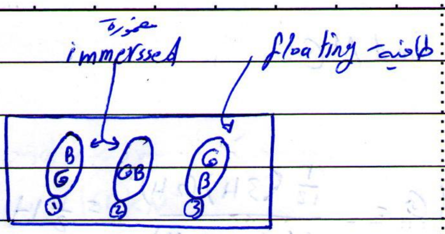
The fraction of the volume above the mercury $= 1 - 0.534$
$= 0.466$

Subject Date No.

* stability :-

B → center of [redacted] bouyancy

G → center of gravity




① stable ② natural ③ unstable

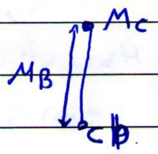
③ ⟹ unstable, stable يمكن أن تكون

وهذا يعتمد على ال metacenter

* metacenter :- is defined as the point of inter section of lines action of the bouyancy force befor and after rotation.

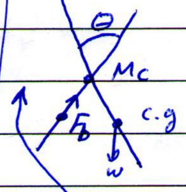
* meta center height ($MG$) : is defind as the distance between meta center (m) and center of gravity.

$$y_{MC} = y_{CB} + M_B$$




$y_{c.g} > y_{Mc}$ unstable

$y_{c.g} < y_{Mc}$ stable.



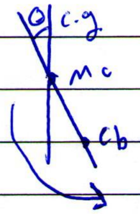

c.g below Mc

stable

Right couple.

$$MG = \frac{I_{oo}}{V_{dis}} - MB$$

$I_{oo}$ ⟹ العزم على المحور




unstable

$MG \rightarrow +ve \Rightarrow$ stable

$MG \rightarrow -ve \Rightarrow$ unstable

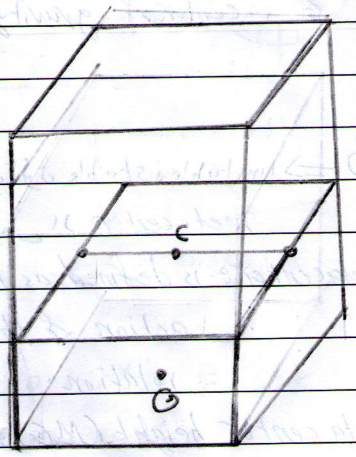
Subject Date No.

ex) find $MG$ :-

$$MG = \frac{\frac{1}{12}(3H)(2H)^3}{H(2H)(3H)} = \frac{1}{2}H$$

$$= \frac{H}{3} - \frac{H}{2} = -ve$$

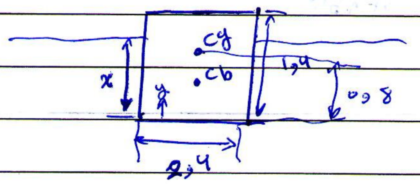
unstable




C → تقع في منتصف الجسم

G → تقع في منتصف الجسم المغمور .

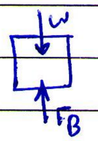
example :-




width = 6m

$w = 150\,kN$

① Find if the body float in the fresh water?




$F_B = w$

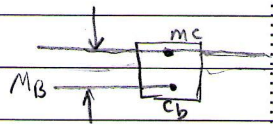
$\gamma_F \forall_d = w \Rightarrow \forall_d = \frac{w}{\gamma_F}$

$x A = \frac{w}{\gamma_F} \Rightarrow x = \frac{w}{\gamma_F \cdot A}$

$x = \frac{150 * 10^3}{9810 * (2,4 * 6)} = 1,06\,m$ مغمور جزئياً (الجسم طافي)

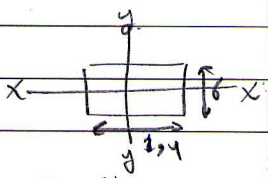
② where is the center of ~~buoyancy~~ buoyancy Force?

مركز الطفو من القاع $\Leftarrow y_{c.b} = \frac{x}{2} = \frac{1,06}{2} = 0,53\,m$




$M_b = \frac{I}{\forall d}$

$y_{mc} = y_{cp} + M_B$




$M_b = \frac{I_y}{\forall_d} = \frac{\frac{6 * (2,4)^3}{12}}{1,06 * (6 * 2,4)}$

$= 0,45\,m$

نجيب أقل I

$I_y < I_x$

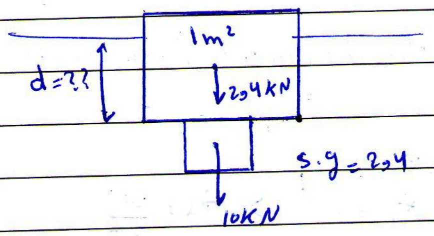
Subject Date No.

$y_{mc} = y_{cb} + M_b = 0{,}53 + 0{,}45 = \underline{0{,}98\,m}$

stable

$y_{mc} > y_{c.g}$ is stable

example :-




sol :-

$F_B = W$ | $W = 10 + 2{,}4 = 12{,}4\ kN$

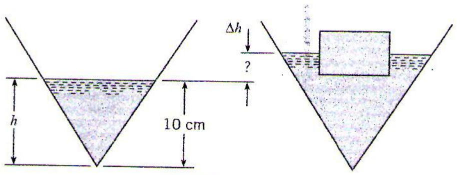
$F_B = \gamma \forall_D$ | $F_B = \gamma \forall_D$

$12{,}4 = 9{,}81\,(0{,}425 + (1 * d * 1))$ | $\forall_D = \dfrac{10}{9{,}81 * 2{,}4} = 0{,}425\,m^3$

$\Rightarrow d = 0{,}84\,m$

Subject buoyancy and stability Problem Date No. ☆

**3.90** A 90° inverted cone contains water as shown. The volume of the water in the cone is given by $\forall = (\pi/3)h^3$. The original depth of the water is 10 cm. A block with a volume of 200 cm$^3$ and a specific gravity of 0.6 is floated in the water. What will be the change (in cm) in water surface height in the cone?




PROBLEM 3.90

$\forall = \frac{\pi}{3} h^3$

$\forall_{block} = 200\ cm^3$

$s.g = 0{,}6$

$0{,}6 = \frac{\gamma_{block}}{\gamma_{water}}$

$\gamma_{block} = 0{,}6 * \gamma_w$

$W_{displaced\ water} = W_{block}$

$\forall_w \gamma_w = \forall_b \gamma_b$

$\forall_w = \frac{\gamma_b}{\gamma_w} \forall_b \Rightarrow \forall_w = 0{,}6\ \forall_b$

$= 0{,}6 * 200 = 120\ cm^3$

$\forall_0 = \frac{\pi}{3} (10\ cm)^3 = 1047{,}2\ cm^3$

$\forall_{Final} = 120 + 1047{,}2 = 1167{,}2\ cm^3$

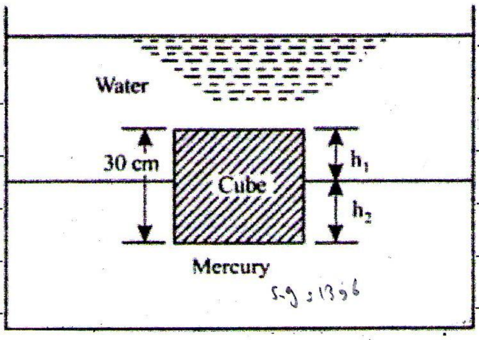
$\frac{\pi}{3} h^3 = 1167{,}2$

$h = 10{,}368\ cm$

$\Delta h = 10{,}368 - 10 = 0{,}368\ cm$

Subject Date No.

**Example 4.4.** *A metallic cube 30 cm side and weighing 450 N is lowered into a tank containing a two-fluid layer of water and mercury. Determine the position of block at mercury-water interface when it has reached equilibrium.*




$F_b = W$

$450 = F_{bw} + F_{bHg}$

$450 = 9810(h_1 \times 0.3 \times 0.3)$
$+ (9810 \times 13.6)(h_2 \times 0.3 \times 0.3)$

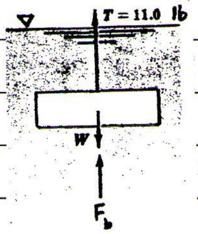
$(h_1 + 13.6 h_2) = \frac{450}{9810 \times 0.3 \times 0.3} = 0.509 \text{ m}$

$h_1 + h_2 = 0.3$
$h_1 + 13.6 h_2 = 0.509$

$h_2 = 0.01658 \text{ m}$
$h_1 = 0.2834 \text{ m}$

Subject Date No.

A prismatic object 8 in thick by 8 in wide by 16 in long is weighed in water at a depth of 20 in and found to weigh 11.0 lb. What is its weight in air and its specific gravity?




$\Sigma F_y = 0$

$T + F_b - W = 0$

$F_b = W_{dis}$

$\gamma \forall$

$F_b = 62.4 \left( \frac{(8)(8)(16)}{1728} \right) = 37 \text{ lb}$

$11 + 37 - W = 0 \Rightarrow W = 48 \text{ lb}$

$s.g = \frac{48}{37} = \boxed{1.3}$

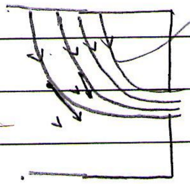
N O T E B O O K

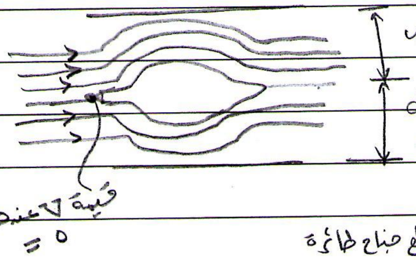
Subject Ch4:- Flowing Fluid and pressure ~~vs~~ variation Date No.

* stream line :- هي خطوط ترسم في مسار الflow المتحرك لمعرفة اتجاه ال Flow

* velocity vector $\xrightarrow[\text{Tangent}]{}$ stream line

* stream line Group $\Rightarrow$ flow patternes




مقطع جناح طائرة

Aeroflow cross section

* الضغط في الأعلى أقل من الأسفل مما يؤدي إلى رفع الجناح للأعلى

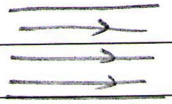
* Velocity :-

$$\vec{V} = F(s, t)$$

Time

displacement

(الإزاحة ، قد يكون بعد أو بعدين أو ٣ ....)



uniform

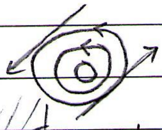
يجب أن تكون stream line Linear + parallel

* uniform flow :- التدفق المنتظم

$\frac{dV}{ds} = 0 \Rightarrow$ السرعة ثابتة مع الإزاحة .

* non uniform flow :-

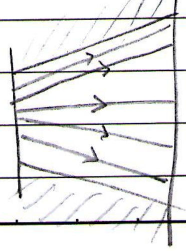
$\frac{dV}{ds} \neq 0$



المسافة متساوية

(لا اتجاه الماس متغير)

non uniform




non-uniform

Subject Date No.

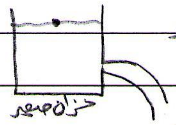
* steady flow :- مستقر لا علاقة بالزمن

$$\frac{dV}{dt} = 0$$

* unsteady flow :- غير مستقر

$$\frac{dV}{dt} \neq 0$$ السرعة تتغير مع الزمن

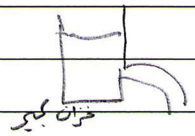
*عملية فتح الصمام ⇐ unsteady




$V = \sqrt{2gH}$

السرعة لها علاقة مع الزمن

∴ السرعة تتغير ⇐ unsteady flow




steady

* نسأل هل تتغير العناصر مع الزمن؟

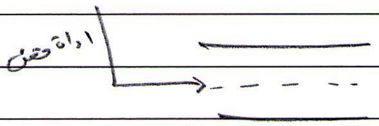
* path lines :- خطوط المسارات

(المسار الذي يتحرك فيه الـ flow من فترة زمنية)

* stream line :- خطوط تصف حركة الـ flow عند لحظة ما

* path line :- المسار الذي يتخذه الـ flow خلال فترة زمنية وليس عند لحظة

* streak lines :- عكس اتجاه path line



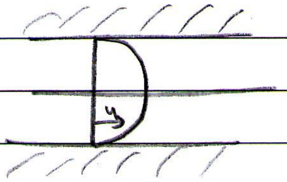

$\wedge \Rightarrow$ path line

$\vee \Rightarrow$ stream line

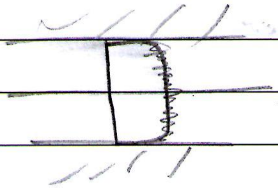
Subject                    Date                    No.

* Laminar flow:- ال Flow عبارة عن طبقات تتحرك مع بعضها دون الانتقال إلى الصفيحة الأعلى والأقل منها
( Flow صفائحي )

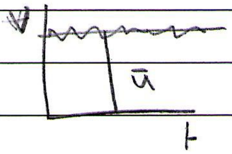
* Turbulent flow:- تداخل جزيئات ال Flow ( تنتقل من طبقة إلى أخرى )




Flow داخل Pipe velocity profile ⇒ laminar Flow (steady)



unsteady ⇒ Turbulent flow




السرعة تتزايد وتتناقص

$$u = \bar{u} + u'$$

$\bar{u}$: قيمة ثابتة مع الزمن

$u'$: قيمة متغيرة مع الزمن (متردد)

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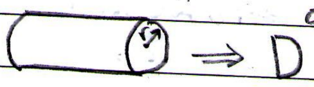


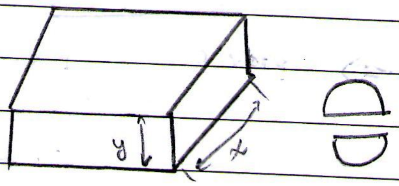
one dimensional flow

السرعة تتغير باتجاه r

r ، θ ، z

✓ ✗ ✗

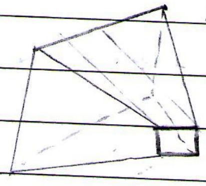
two dimensional flow



x y

✓ ✓

three dimensional flow



x y z

✓ ✓ ✓

** Acceleration:-

$$\vec{V} = v(s, t)$$

$$\vec{a} = \frac{dv}{dt} = \left[\frac{dv}{ds}\frac{ds}{dt} + \frac{dv}{dt}\right]_{et}$$

باتجاه tangent

$$\vec{a}_{\text{tangent thoeram}} = \left[\frac{dv}{ds}\frac{ds}{dt} + \frac{dv}{dt}\right]_{et} + \frac{v^2}{r}_{et}$$

$$V\frac{de}{dt}$$

$$\frac{V}{r}$$

* اذا كان اتجاه السرعة خطي ← لا يوجد $\frac{V^2}{r}$ لأنه لا يوجد r اذن متجه

$V\frac{de}{dt} = 0$

Subject Date No.

$\frac{\partial v}{\partial t} \Rightarrow$ local acceleration تغير السرعة نسبة إلى الزمن

Steady :، إذن السرعة ثابتة مع الزمن $0 = \frac{\partial v}{\partial t}$ إذا كانت قيمة x

$\frac{\partial v}{\partial s} \frac{\partial s}{\partial t} \Rightarrow \frac{\partial v}{\partial s} \times V$

$V \frac{\partial v}{\partial s} \Rightarrow$ convective acceleration

تغير السرعة نسبة الى المسافة

$V \frac{\partial v}{\partial s} = 0 \Rightarrow$ uniform

$V \frac{\partial v}{\partial s} \neq 0 \Rightarrow$ non-uniform

$\frac{V^2}{r} \Rightarrow$ centripetal acceleration (normal acceleration)

$$\vec{V} = \underbrace{u\,i}_{x\ \text{مركبة}} + \underbrace{v\,j}_{y\ \text{مركبة}} + \underbrace{w\,k}_{z\ \text{مركبة}}$$

$$\vec{a} = a_x i + a_y j + a_z k$$

$$a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}$$

$$a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}$$

$$a_z = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} + \frac{\partial w}{\partial t}$$

convective

local

NOTEBOOK

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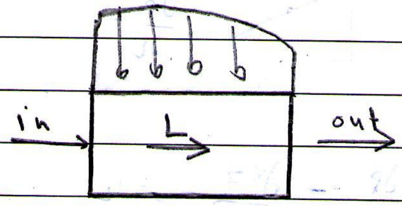
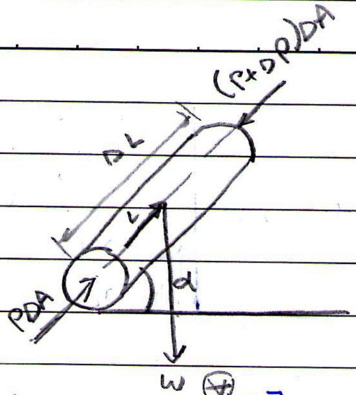
** pressure variation:-

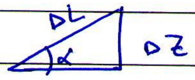



$$p\Delta A - (p+\Delta p)\Delta A - W\sin\alpha = \rho\,[\Delta A\,\Delta L]\,a_L$$

$$-\Delta p\,\Delta A - \gamma\,\Delta A\,\Delta L\sin\alpha = \rho\,\Delta A\,\Delta L\,a_L$$

$$-\frac{\Delta p}{\Delta L} - \gamma\sin\alpha = \rho a_L$$




$\sin\alpha = \frac{\Delta Z}{\Delta L}$

$$-\left(\frac{\Delta p}{\Delta L} + \gamma\frac{\Delta Z}{\Delta L}\right) = \rho a_L$$

$\Delta L \to 0$

$$\Rightarrow -\left(\frac{\partial p}{\partial L} + \frac{\partial z\,\gamma}{\partial L}\right) = \rho a_L$$

Euler's equations of motion of fluid

$$\boxed{-\left[\frac{\partial(p+\gamma z)}{\partial L}\right] = \rho a_L}$$

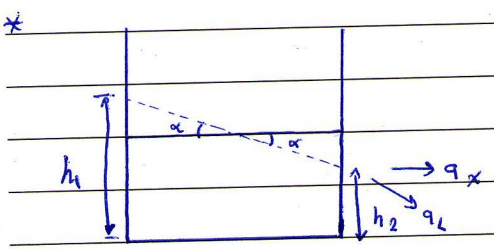
if $a_L = 0 \Rightarrow -\left[\frac{\partial(p+\gamma z)}{\partial L}\right] = 0$

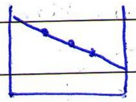
$$\frac{\partial p}{\partial L} = -\gamma\frac{\partial z}{\partial L}$$

$$\boxed{\frac{\partial p}{\partial z} = -\gamma}$$

Subject Date No.

*



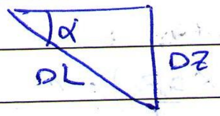



$\frac{\partial p}{\partial L} = 0$

$-\frac{\partial p}{\partial L} - \frac{\partial \gamma z}{\partial L} = \rho a_L$

$\frac{\partial \gamma z}{\partial L} = -\rho a_L$

$\gamma \frac{\partial z}{\partial L} = -\rho a_L = -\rho (a_x \cos\alpha)$



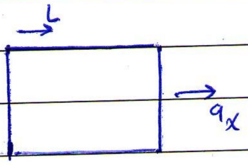

$\sin\alpha = -\frac{DZ}{DL}$

$-\gamma \sin\alpha = -\rho a_x \cos\alpha$

$\frac{\sin\alpha}{\cos\alpha} = \frac{\rho a_\alpha}{\gamma}$

$\boxed{\tan\alpha = \frac{a_x}{g}}$

**




$-\frac{\partial p}{\partial L} - \frac{\partial \gamma z}{\partial L} = \rho a_L$

$-\frac{\partial p}{\partial L} - \frac{\gamma \partial z}{\partial L} = \rho a L$

$\Downarrow$

zero لان Z ثابتة بالنسبة الى L

$-\frac{\partial p}{\partial L} = \rho a_L = \rho a_x \Rightarrow dp = -\rho a_x \, dL$

$\boxed{p = -\rho a_\alpha L + C}$

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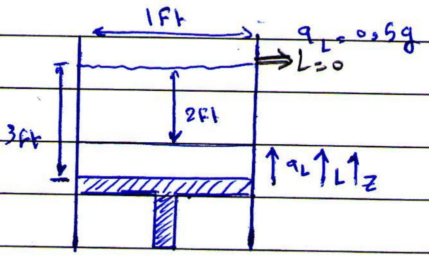



* what is the value pressure at two fit depth?

$$Z = L \Rightarrow \boxed{\frac{\partial z}{\partial L} = 1}$$

$$-\frac{\partial p}{\partial L} - \frac{\partial z}{\partial L}\gamma = \rho a_L$$

$$-\frac{\partial p}{\partial L} - \gamma = \rho a_L \Rightarrow -\frac{\partial p}{\partial L} = \rho a_L + \gamma$$

$$\frac{\partial p}{\partial L} = -(\rho a_L + \gamma) \Longrightarrow p = -(\rho a_L + \gamma)L + c$$

$$p = -1,5\gamma L + c$$

$$\boxed{\frac{\rho g}{2} + \gamma = 1,5\gamma}$$

at $L = 0 \Rightarrow p = 0$

$$0 = -1,5\gamma(0) + c \Rightarrow c = 0$$

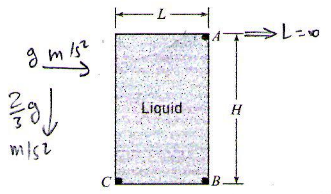
$$\boxed{p = -1,5\gamma L}$$

$$P(-2) = -1,5 * (62,4) * (-2) = 3 * 62,4$$

$$= 187,2 \ lbf/ft^2$$

Subject ✱ Date No.

**4.38** The closed tank shown, which is full of liquid, is accelerated downward at $\frac{2}{3}g$ and to the right at $1g$. Here $L = 2.5$ m, $H = 3$ m, and the liquid has a specific gravity of 1.3. Determine $p_C - p_A$ and $p_B - p_A$.




① $(P_C - P_A) = ??$ $\quad (P_B - P_A) + (P_C - P_B)$

Ⓐ $(P_B - P_A) = ??$

$$\frac{\partial}{\partial L}(P + \gamma z) = -\rho a$$

$$\frac{\partial P}{\partial L} + \gamma = -\rho a \qquad \Big| \; \frac{dz}{\partial L} = 1$$

$$\frac{\partial P}{\partial L} = -(\gamma + \rho a)$$

$$\frac{dP}{dL} = -(\gamma + \rho(\tfrac{-2}{3}g))$$

$$\frac{\partial P}{\partial L} = \frac{-\gamma}{3} \Rightarrow \int \frac{\partial P}{\partial L} = \int \frac{-\gamma}{3}\, dL$$

$$\boxed{P = \frac{-\gamma}{3} L + C}$$ ز تقل بزيادة L

at $L = 0 \Rightarrow P = P_A$

$$P_A = -\frac{\gamma}{3}(0) + C$$

$$\Rightarrow C = P_A$$

$$P_B = \frac{-\gamma}{3}(-3) + P_A$$

$$P_B = \gamma + P_A$$

$$\boxed{P_B - P_A = \gamma}$$

$$\left(\frac{\partial p}{\partial L} + \gamma \frac{\partial z}{\partial L}\right) = -\rho a$$

لا تتغير Z مع L

(قيمة Z ثابتة مع L)

$$\int \frac{\partial p}{\partial L} = \int -\rho a$$

$$p = -\rho a L + c$$

at $L = 0 \Rightarrow p = p_B$

$$p = -\rho a L + c$$

$$\boxed{c = p_B}$$

$$p = -\rho g + p_B = -\gamma L + p_B$$

$$p_C = -\gamma(-2,5) + p_B$$

$$p_C - p_B = 2,5\gamma$$

$$p_C - p_A = (p_C - p_B) + (p_B - p_A)$$

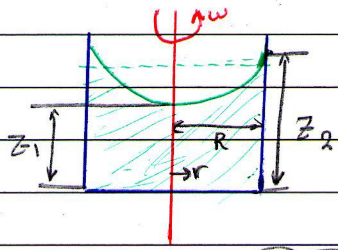
$$= 2,5\gamma + \gamma = 3,5\gamma$$

$$= 3,5 * \underbrace{1,3 * 9,81}_{kN/m^2}$$

$$= 44,63 \text{ kpa}$$

Subject Date No.

## * Rotation of tank :-




$$\frac{d}{dr}(p+\gamma z) = -\rho a_n$$

$$\frac{d}{dr}(p+\gamma z) = -\rho\left(-\frac{v^2}{r}\right)$$

$a_n = \frac{v^2}{r}$ $v = wr$

لما اتجاه r عكس الـ center للمحيط

إشارة السالب لأن الـ normal باتجاه الـ center

$$\frac{d(p+\gamma z)}{dr} = \rho\left(\frac{w^2 r^2}{r}\right) = \rho w^2 r$$

$$d(p+\gamma z) = (\rho w^2 r)\,dr$$

$$p+\gamma z = \frac{\rho w^2 r^2}{2} + c$$

$$\boxed{p+\gamma z - \frac{\rho w^2 r^2}{2} = c}$$ ⇒ عند أي r القيمة ثابتة

$r_{out}$ أو $r=0$

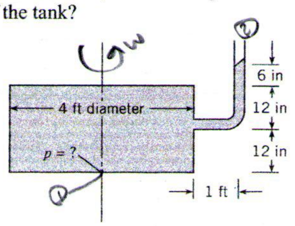
$$\cancel{p_1} + \gamma z_1 - \cancel{\frac{\rho w^2 r_1^2}{2}} = \cancel{p_2} + \gamma z_2 - \frac{\rho w^2 r_2^2}{2}$$

$p_1 = p_2 = 0$

مفتوح للضغط الجوي

Subject Date No.

**4.41** This closed tank, which is 4 ft in diameter, is filled with water ($\gamma = 62.4\ lbm/ft^3$) and is spun around its vertical centroidal axis at a rate of 15 rad/s. An open piezometer is connected to the tank as shown so that it is also rotating with the tank. For these conditions, what is the pressure at the center of the bottom of the tank?




PROBLEM 4.41

$r_1 = 0$

$r_2 = 3ft$

$P_2 = 0$

$z_1 = 0$

$z_2 = 30in / 12 = 2,5ft$

$\rho = 1,94$

$$P_1 + \cancel{\gamma z_1} - \cancel{\frac{\rho w^2 r_1^2}{2}} = \cancel{P_2} + \gamma z_2 - \frac{\rho w^2 r_2^2}{2}$$

$$P_1 = \gamma z_2 - \frac{\rho w^2 r_2^2}{2}$$

$$= (62,4 \times (2,5)) - \frac{1,94 (15)^2 (3)^2}{2}$$

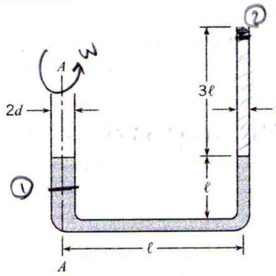
$$= -1808\ psf$$

$$= -1808\ 1ft^3/144\ in^3$$

$$= -12,557\ psi$$

Subject Date No.

**4.50** Water stands in these tubes as shown when no rotation occurs. Derive a formula for the angular speed at which water will just begin to spill out of the small tube when the entire system is rotated about axis $A$-$A$.




PROBLEM 4.50

المطلوب أقصى w حتى يخرج ال Flow
من الانبوبة؟

$r_1 = 0$

$r_2 = L$

$z_1 = ??$

$Z_2 = 4L$

$P_1 = P_2$

$$P_1 + \gamma Z_1 - \frac{\rho w^2 r_1^2}{2} = P_2 + \gamma Z_2 - \frac{\rho w^2 r_2^2}{2}$$

$$\gamma Z_1 = 4L\gamma - \frac{\rho w^2 L^2}{2}$$

$$w = \sqrt{2\left(\frac{4L\gamma - \gamma Z_1}{\rho L}\right)} = \sqrt{\frac{g(8L - Z_1)}{L^2}}$$

$$w = \frac{1}{L}\sqrt{g(8L - 2Z_1)}$$

$V_1 = V_2$

before Rotation, After Rotation

$LA_1 + LA_2$

$= Z_1 A_1 + Z_2 A_2$

$\Rightarrow \boxed{Z_1 = \frac{L}{4}}$

$$w = \frac{1}{L}\sqrt{g(8L - 2Z_1)}$$

$$w = \frac{1}{L}\sqrt{7.5\, gL}$$

Subject Date No.

* **Bernouli equation :-**

$$\frac{d}{dL}(P+\gamma Z) = -\rho a_t \Rightarrow \text{Euler equation}$$

$$a_t = v\frac{dv}{ds} + \frac{dv}{dt} \rightarrow \text{steady}$$ تتغير السرعة مع الزمن

$$a_t = v\frac{dv}{ds} \Rightarrow v\frac{dv}{ds} = \frac{d\frac{v^2}{2}}{ds} \qquad \frac{d\frac{v^2}{2}}{ds} = \frac{2}{2}\,v\frac{dv}{ds}$$

$$\frac{d}{ds}(P+\gamma Z) = -\rho v\frac{dv}{ds} = -\rho\frac{d}{ds}\left(\frac{v^2}{2}\right)$$

$$= \frac{-d}{ds}\left(\rho\frac{v^2}{2}\right)$$

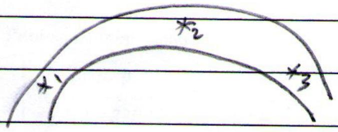
$$\frac{d}{ds}\left(P+\gamma Z+\rho\frac{v^2}{2}\right) = 0$$

$$\boxed{\underbrace{P}_{\text{static pressure}} + \underbrace{\gamma Z}_{} + \underbrace{\rho\frac{v^2}{2}}_{\text{dynamic pressure}} = C}$$

static pressure — pizometer pressure — dynamic pressure

$$\div\gamma \Rightarrow \underbrace{\frac{P}{\gamma}+Z}_{\text{pizometric head}} + \frac{V^2}{2g} = C$$

pizometric head ارتفاع



Bernoulli equation $$\boxed{\frac{P_1}{\gamma}+Z_1+\frac{V_1^2}{2g} = \frac{P_2}{\gamma}+Z_2+\frac{V_2^2}{2g} = \frac{P_3}{\gamma}+Z_3+\frac{V_3^3}{2g}}$$

اذا زادت P تقل V والعكس صحيح

NOTEBOOK

Subject Date No.

* bernouli For gas $\Rightarrow$ $P + \frac{\rho V^2}{2} = C$

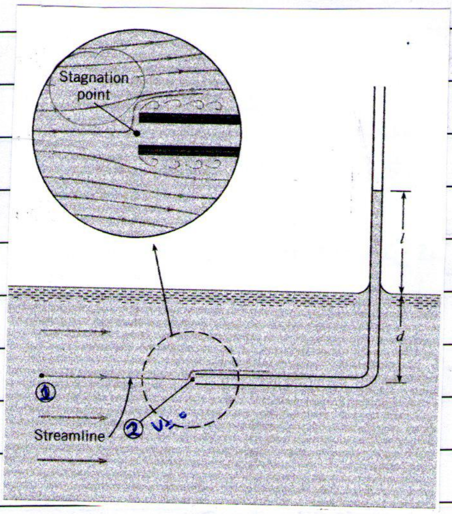
$$\frac{P}{\gamma} + \frac{V^2}{2g} = C$$

يهمل الارتفاع لان تأثيره قليل خلال الغازات

translation motion $\Rightarrow$ irrotation flow + steady flow

## ** bernouli Application :-

## ** stagnation tube :-




عند ② السرعة صفر لان السائل ارتفع ووقف

$$\frac{P_1}{\gamma} + \cancel{Z_1} + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + \cancel{Z_2} + \cancel{\frac{V_2^2}{2g}}$$

$$V_1 = \sqrt{\frac{2g(P_2 - P_1)}{\gamma}}$$

$$P_1 = \gamma d$$

$$P_2 = (\gamma d + \gamma L)$$

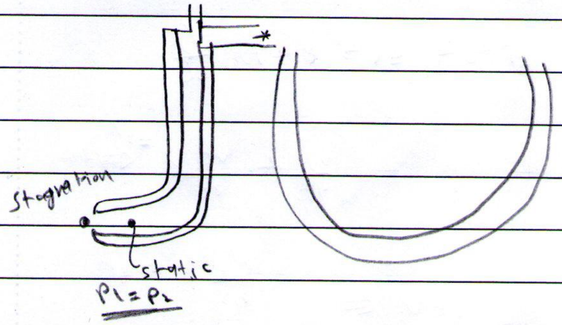
$$P_2 - P_1 = \gamma L$$

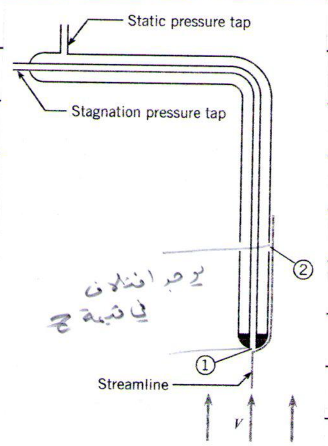
$$V_1 = \sqrt{2gL}$$

Poletechnic Lecture Note

Subject Date No.

**** Pitot tube :-**




$$V_1 = \sqrt{2g\left(\frac{P_2 - P_1}{\gamma} + (z_2 - z_1)\right)}$$

**EXAMPLE 4.9 PITOT TUBE APPLICATION WITH PRESSURE GAGE**

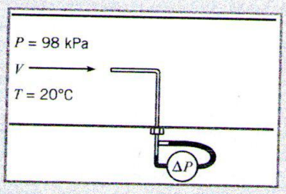
A differential pressure gage is connected across the taps of a Pitot-static tube. When this Pitot-static tube is used in a wind tunnel test, the gage indicates a $\Delta p$ of 730 Pa. What is the air velocity in the tunnel? The pressure and temperature in the tunnel are 98 kPa absolute and 20°C, respectively.

***Problem Definition***

**Situation:** Differential pressure gage attached to Pitot-static tube for velocity measurement in wind tunnel.

**Find:** Air velocity (in m/s).

**Sketch:**




**Assumptions:**

1. Airflow is steady.
2. Pitot-tube equation applicable.

**Properties:** Table A.2, $R_{air} = 287$ J/kg K.

***Plan***

1. Using the ideal gas law, calculate air density.
2. Using the Pitot-static tube equation, calculate the velocity.

***Solution***

1. Density calculation:

$$\rho = \frac{p}{RT} = \frac{98 \times 10^3 \text{ N/m}^2}{(287 \text{ J/kg K}) \times (20 + 273 \text{ K})} = 1.17 \text{ kg/m}^3$$

2. Pitot-static tube equation with differential pressure gage:

$$V = \sqrt{2\Delta p/\rho}$$

$$V = \sqrt{(2 \times 730 \text{ N/m}^2)/(1.17 \text{ kg/m}^3)} = \boxed{35.3 \text{ m/s}}$$

N O T E B O O K

Subject Date No.

* Concept of Rotation :-

$$\boxed{\vec{\Omega} = \Omega_x i + \Omega_y j + \Omega_z k} \begin{cases} \neq 0 \Rightarrow \text{rotational flow} \\ = 0 \Rightarrow \text{irrotational flow} \end{cases}$$

$$\vec{\Omega} = \frac{1}{2} \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix}$$

$$\Omega_x = \frac{1}{2}\left(\frac{\partial}{\partial y} w - \frac{\partial}{\partial z} v\right)$$

$$\Omega_y = \ominus \frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right)$$

clockwise

$$\Omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$$

* vorticity = $2\vec{\Omega} = \nabla * \vec{V}$

$$\nabla = \frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$$

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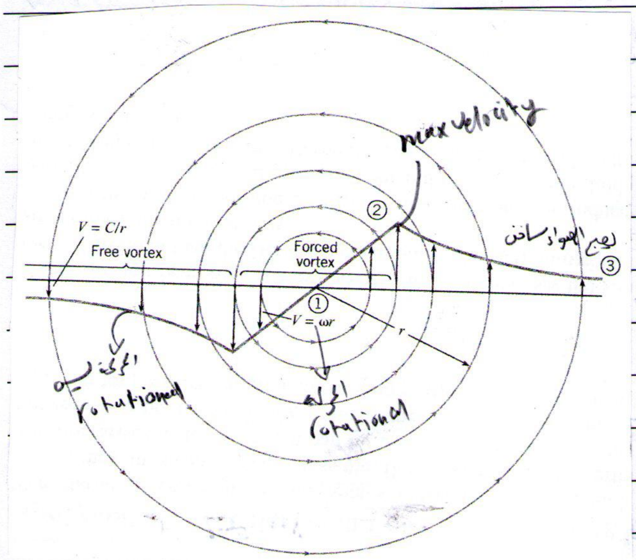
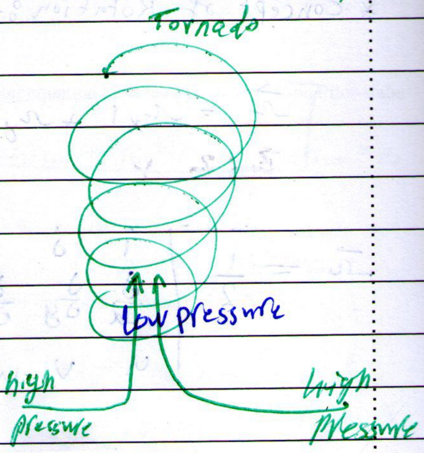
## ** cyclonic pressure variation :-




* في الوسط مركز زوبعة تتزايد السرعة
كلما ابتعدنا عن مركز العاصفة
ان نصل الى max. velocity

$(2 \to 3) \Rightarrow$ irrotational flow

$$\frac{P_3}{\gamma} + Z_3 + \cancel{\frac{V_3^2}{2g}} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

$V_3 = 0$

$P_3 = P_a$

$$\frac{V_2^2}{2g} = \frac{P_a}{\gamma} - \frac{P_2}{\gamma} + (Z_3 - Z_2)$$

(لا تغيّر Z لأن $\gamma$ للهواء قليلة)
مع V و P

$$P_2 = P_a - \left(\frac{V_2^2}{2g} + Z_3 - Z_2\right)\gamma$$

$$P_2 = P_a - \rho \frac{V_2^2}{2}$$

$$\Rightarrow \boxed{V_2 = \sqrt{2\left(\frac{P_a - P_2}{\rho}\right)}}$$

max velocity

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$(1 \to 2) \Rightarrow$

$$\frac{P_1}{\gamma} + \cancel{Z_1} - \frac{\rho w^2 r_1^2}{2g} = \frac{P_2}{\gamma} + \cancel{Z_2} - \frac{w_2^2 r_2^2}{2g}$$

$Z_1$ و $Z_2$ تهمل

$r_1 = 0$

$$\times \gamma \Rightarrow \boxed{P_1 = P_2 - \rho \frac{V_2^2}{2}}$$

$$P_1 = P_a - \frac{\rho V^2}{2} - \frac{\rho V_2^2}{2} \quad \Bigg| \quad P_2 = P_a - \frac{V_2^2}{2}$$

$$\boxed{P_1 = P_a - \rho V_2^2}$$

** pressure Coefficient :- $(C_p)$ (تحديد مقدار القوى المؤثرة على أي جسم)

$$C_p = \frac{P - P_0}{\frac{1}{2} \rho V_0^2}$$

$$C_p = \frac{\frac{P - P_0}{\gamma}}{\frac{V^2}{2g}} = \frac{P - P_0}{\frac{1}{2} \rho V_0^2}$$

قوى Fluid

drag ← نتيجة احتكاك (قوى ممانعة السفن)

left ← الفرق بين القوى العمودية على الطائرة وجناحها للأسفل والأعلى (ناتج عن فرق الضغط بين الأعلى والأسفل مما يؤدي إلى ارتفاع الطائرة)

N O T E B O O K

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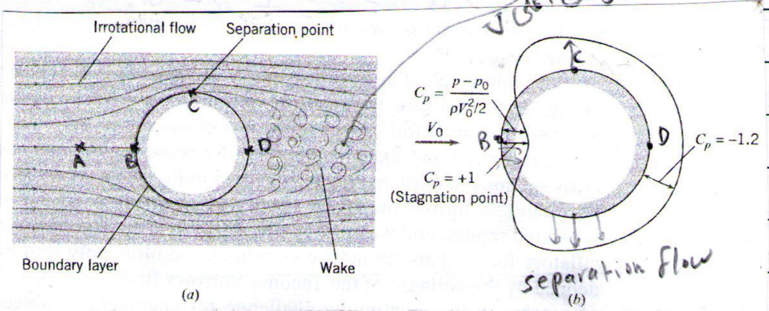
## * Flow over cylinder :-

انخفاض في P يؤدي
الى الاحتكاك ولن يعكس الى اعلى




* داخل الاسطوانة موجب
وخارج الاسطوانة سالب
* يقاس التغير نسبة
الى حدود الاسطوانة

B ⇒ stagnation point

at B ⇒ $CP = 1$

at C ⇒ $CP = -3$

* برنولي A, B

$P_0 \Rightarrow P_A$

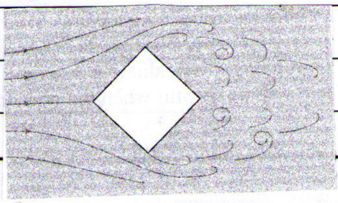
$$\frac{P_0}{\gamma} + \frac{V_0^2}{2g} + \cancel{Z_A} = \frac{P_B}{\gamma} + \cancel{\frac{V_B^2}{2g}} + \cancel{Z_B}$$

$$\frac{P_B - P_0}{\gamma} = \frac{V_0^2}{2g}$$

$$P_B - P_0 = \frac{\rho V_0^2}{2} \quad \therefore \quad \boxed{CP = 1}$$

$(B \to C) \Longrightarrow$ يحدث acceleration لل Flow ، يؤدي الى
الى زيادة سرعة وانخفاض الضغط

$(C \to D) \Longrightarrow$ deacceleration ، (B ، C) يحدث عكس
انخفاض في السرعة
وزيادة في الضغط



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**EXAMPLE 4.1 EVALUATING ACCELERATION IN A NOZZLE**

A nozzle is designed such that the velocity in the nozzle varies as

$$u(x) = \frac{u_0}{1.0 - 0.5x/L}$$

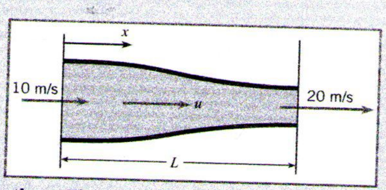
where the velocity $u_0$ is the entrance velocity and $L$ is the nozzle length. The entrance velocity is 10 m/s, and the length is 0.5 m. The velocity is uniform across each section. Find the acceleration at the station halfway through the nozzle ($x/L = 0.5$).

*Problem Definition*

**Situation:** Given velocity distribution in a nozzle.

**Find:** Acceleration at nozzle midpoint.

**Sketch:**




**Assumptions:** Flow field is quasi–one-dimensional (negligible velocity normal to nozzle centerline).

*Plan*

1. Select the pathline along the centerline of the nozzle.
2. Evaluate the convective, local, and centripetal accelerations in Eq. (4.5).
3. Calculate the acceleration.

1- Evaluation of terms :-

* convective acceleration :-

$$\frac{\partial u}{\partial x} = \frac{-u_0}{\left(1-\frac{0.5x}{L}\right)^2} * \left(\frac{-0.5}{L}\right)$$

$$= \frac{1}{L}\,\frac{0.5\,u_0}{\left(1-\frac{0.5x}{L}\right)^2}$$

$$u\frac{\partial u}{\partial x} = 0.5\,\frac{u_0^2}{L}\,\frac{1}{\left(1-\frac{0.5x}{L}\right)^3}$$

evaluation at $\frac{x}{L} = 0.5$ :-

$$u\frac{\partial u}{\partial x} = 0.5 * \frac{10^2}{0.5} * \frac{1}{0.75^3}$$

$$= 237\ m/s^2$$

* local acceleration :-

$$\frac{\partial u}{\partial t} = 0$$

* centripetal acceleration :-

$$\frac{u^2}{r} = 0$$

2- Acceleration :-

$$a_x = 237\ m/s^2 + 0 = 237\ m/s^2$$

$a_n$ (normal to pathline) $= 0$

**EXAMPLE 4.2 APPLICATION OF EULER'S EQUATION TO ACCELERATION OF A FLUID**

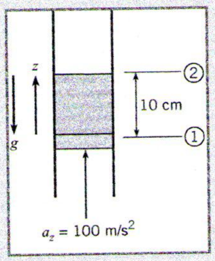
A column water in a vertical tube is being accelerated by a piston in the vertical direction at 100 $m/s^2$. The depth of the water column is 10 cm. Find the gage pressure on the piston. The water density is $10^3$ $kg/m^3$.

*Problem Definition*

**Situation:** A column of water is being accelerated by a piston.

**Find:** The gage pressure on the piston.

**Sketch:**




**Assumptions:**

1. Acceleration is constant.
2. Viscous effects are unimportant.
3. Water is incompressible.

**Properties:** $\rho = 10^3$ kg/m$^3$

*Plan*

1. Apply Euler's equation, Eq. (4.8), in the $z$-direction.
2. Integrate equation and apply limits at sections 1 and 2.
3. Set pressure equal to zero (gage pressure) at cross-section 2 (atmosphere).
4. Calculate the pressure on piston (cross-section 1).

*Solution*

1. Because the acceleration is constant there is no dependence on time so the partial derivative in Euler's equation can be replaced by an ordinary derivative. Euler's equation in $z$-direction:

$$\frac{d}{dz}(p + \gamma z) = -\rho a_z$$

2. Integration between sections 1 and 2:

$$\int_1^2 d(p + \gamma z) = \int_1^2 (-\rho a_z) dz$$

$$(p_2 + \gamma z_2) - (p_1 + \gamma z_1) = -\rho a_z (z_2 - z_1)$$

3. Substitution of limits:

$$p_1 = (\gamma + \rho a_z)\Delta z = \rho(g + a_z)\Delta z$$

4. Evaluation of pressure:

$$p_1 = 10^3 \text{ kg/m}^3 \times (9.81 + 100) \text{ m/s}^2 \times 0.1 \text{ m}$$

$$p_1 = \boxed{10.9 \times 10^3 \text{ Pa} = 10.9 \text{ kPa, gage}}$$

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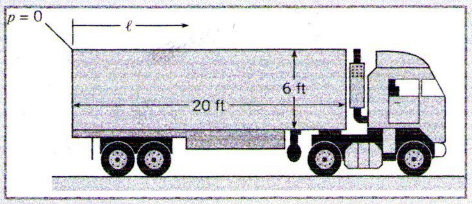
EXAMPLE 4.3 PRESSURE IN A DECELERATING TANK OF LIQUID

The tank on a trailer truck is filled completely with gasoline, which has a specific weight of 42 lbf/ft$^3$ (6.60 kN/m$^3$). The truck is decelerating at a rate of 10 ft/s$^2$ (3.05 m/s$^2$).

a. If the tank on the trailer is 20 ft (6.1 m) long and if the pressure at the top rear end of the tank is atmospheric, what is the pressure at the top front?

b. If the tank is 6 ft (1.83 m) high, what is the maximum pressure in the tank?

**Sketch:**




*Problem Definition*

**Situation:** Decelerating tank of gasoline with pressure equal to zero gage at top rear end.

**Find:**

1. Pressure (psfg and kPa, gage) at top front of tank.
2. Maximum pressure (psfg and kPa, gage) in tank.

**Assumptions:**

1. Deceleration is constant.
2. Gasoline is incompressible.

**Properties:** $\gamma = 42$ lbf/ft$^3$ (6.60 kN/m$^3$)

*Plan*

1. Apply Euler's equation, Eq. (4.8), along top of tank. Elevation, $z$, is constant.
2. Evaluate pressure at top front.
3. Maximum pressure will be at front bottom. Apply Euler's equation from top to bottom at front of tank.
4. Using result from step 2, evaluate pressure at front bottom.

*Solution*

1. Euler's equation along the top of the tank

$$\frac{dp}{d\ell} = -\rho a_\ell$$

Integration from back (1) to front (2)

$$p_2 - p_1 = -\rho a_\ell \Delta\ell = -\frac{\gamma}{g} a_\ell \Delta\ell$$

2. Evaluation of $p_2$ with $p_1 = 0$

$$p_2 = -\left(\frac{42 \text{ lbf/ft}^3}{32.2 \text{ ft/s}^2}\right) \times (-10 \text{ ft/s}^2) \times 20 \text{ ft}$$

$$= \boxed{261 \text{ psfg}}$$

In SI units

$$p_2 = -\left(\frac{6.60 \text{ kN/m}^3}{9.81 \text{ m/s}^2}\right) \times (-3.05 \text{ m/s}^2) \times 6.1 \text{ m}$$

$$= \boxed{12.5 \text{ kPa, gage}}$$

3. Euler's equation in vertical direction

$$\frac{d}{dz}(p + \gamma z) = -\rho a_z$$

4. For vertical direction, $a_z = 0$. Integration from top of tank (2) to bottom (3):

$$p_2 + \gamma z_2 = p_3 + \gamma z_3$$

$$p_3 = p_2 + \gamma(z_2 - z_3)$$

$$p_3 = 261 \text{ lbf/ft}^2 + 42 \text{ lbf/ft}^3 \times 6 \text{ ft} = \boxed{513 \text{ psfg}}$$

In SI units

$$p_3 = 12.5 \text{ kN/m}^2 + 6.6 \text{ kN/m}^3 \times 1.83 \text{ m}$$

$$p_3 = \boxed{24.6 \text{ kPa, gage}}$$

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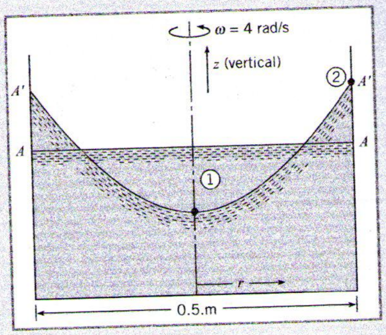
## EXAMPLE 4.4 SURFACE PROFILE OF ROTATING LIQUID

A cylindrical tank of liquid shown in the figure is rotating as a solid body at a rate of 4 rad/s. The tank diameter is 0.5 m. The line $AA$ depicts the liquid surface before rotation, and the line $A'A'$ shows the surface profile after rotation has been established. Find the elevation difference between the liquid at the center and the wall during rotation.

**Sketch:**




*Problem Definition*

**Situation:** Liquid rotating in a cylindrical tank.

**Find:** Elevation difference (in meters) between liquid at center and at the wall.

**Assumptions:** Fluid is incompressible.

*Plan*

Pressure at liquid surface is constant (atmospheric).

1. Apply equation for pressure variation in rotating flow, Eq. (4.13a), between points 1 and 2.
2. Evaluate elevation difference.

*Solution*

1. Equation (4.13a) applied between points 1 and 2.

$$\frac{p_1}{\gamma}+z_1-\frac{\omega^2 r_1^2}{2g}=\frac{p_2}{\gamma}+z_2-\frac{\omega^2 r_2^2}{2g}$$

The pressure at both points is atmospheric, so $p_1 = p_2$ and the pressure terms cancel out. At point 1, $r_1 = 0$, and at point 2, $r = r_2$. The equation reduces to

$$z_2-\frac{\omega^2 r_2^2}{2g}=z_1$$

$$z_2-z_1=\frac{\omega^2 r_2^2}{2g}$$

2. Evaluation of elevation difference:

$$z_2-z_1=\frac{(4\text{ rad/s})^2\times(0.25\text{ m})^2}{2\times 9.81\text{ m/s}^2}$$

$$=\boxed{0.051\text{ m or }5.1\text{ cm}}$$

*Review*

Notice that the surface profile is parabolic.

## EXAMPLE 4.5 ROTATING MANOMETER TUBE

When the U-tube is not rotated, the water stands in the tube as shown. If the tube is rotated about the eccentric axis at a rate of 8 rad/s, what are the new levels of water in the tube?

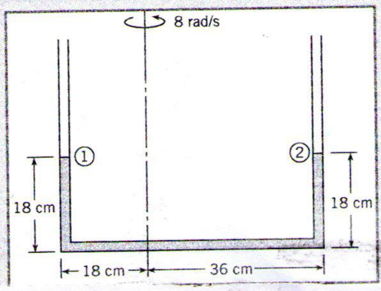
*Problem Definition*

**Situation:** Manometer tube is rotated around an eccentric axis.

**Find:** Levels of water in each leg.

**Assumptions:** Liquid is incompressible.

**Sketch:**




1. Apply the equation for pressure variation in rotating flows, Eq. (4.13a), to evaluate difference in elevation in each leg.
2. Using constraint of total liquid length, find the level in each leg.

*Solution*

1. Application of Eq. (4.13a) between top of leg on left (1) and on right (2):

$$z_1-\frac{r_1^2\omega^2}{2g}=z_2-\frac{r_2^2\omega^2}{2g}$$

$$z_2-z_1=\frac{\omega^2}{2g}(r_2^2-r_1^2)$$

$$=\frac{(8\text{ rad/s})^2}{2\times 9.81\text{ m/s}^2}(0.36^2\text{ m}^2-0.18^2\text{ m}^2)=0.317\text{ m}$$

2. The sum of the heights in each leg is 36 cm.

$$z_2+z_1=0.36\text{ m}$$

Solution for the leg heights:

$$z_2=0.338\text{ m}$$

$$z_1=0.022\text{ m}$$

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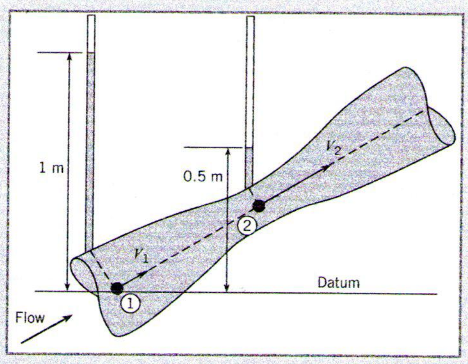
EXAMPLE 4.6 VELOCITY IN A VENTURI SECTION

Piezometric tubes are tapped into a venturi section as shown in the figure. The liquid is incompressible. The upstream piezometric head is 1 m, and the piezometric head at the throat is 0.5 m. The velocity in the throat section is twice large as in the approach section. Find the velocity in the throat section.

**Sketch:**




*Problem Definition*

**Situation:** Incompressible flow in venturi section. Piezometric heads and velocity ratio given.

**Find:** Velocity (in m/s) in venturi section.

**Assumptions:** Viscosity effects are negligible, and the Bernoulli equation is applicable.

*Plan*

1. Write out the Bernoulli equation, Eq. (4.18b), incorporating velocity ratio and solve for throat velocity.
2. Substitute in piezometric heads to calculate throat velocity.

*Solution*

1. The Bernoulli equation with $V_2 = 2V_1$ gives

$$h_1 - h_2 = \frac{V_2^2 - V_1^2}{2g} = \frac{3V_1^2}{2g}$$

$$V_1^2 = \frac{2g}{3}(h_1 - h_2)$$

$$V_2 = 2\sqrt{\frac{2g}{3}(h_1 - h_2)}$$

2. Substitution of values and velocity calculation:

$$V_2 = 2\sqrt{\frac{2 \times 9.81 \text{ m/s}^2 (1 - 0.5) \text{ m}}{3}}$$

$$= \boxed{3.62 \text{ m/s}}$$

*Review*

A piezometric tube could not be used to measure the piezometric head if the pressure anywhere in the line were subatmospheric. In this case, pressure gages or manometers would have to be used.

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EXAMPLE 4.7 OUTLET VELOCITY FROM DRAINING TANK

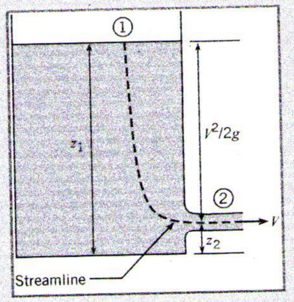
A open tank filled with water and drains through a port at the bottom of the tank. The elevation of the water in the tank is 10 m above the drain. The drain port is at atmospheric pressure. Find the velocity of the liquid in the drain port.

*Problem Definition*

**Situation:** Tank draining through port at bottom.

**Find:** Velocity (in m/s) in drain port.

**Sketch:**




**Assumptions:**

1. Flow is steady.
2. Viscous effects are unimportant.
3. Velocity at liquid surface is much less than velocity in drain port.

*Plan*

Since the flow is steady and viscous effects are unimportant, the Bernoulli equation is applicable along a streamline. The streamline chosen is shown in the sketch with point 1 at the liquid surface and point 2 at the drain port.

1. Apply the Bernoulli equation, Eq. (4.18b), between points 1 and 2.
2. Reduce the equation to yield velocity in drain port.
3. Calculate velocity.

*Solution*

1. The Bernoulli equation between points 1 and 2 on streamline:

$$\frac{p_1}{\gamma}+z_1+\frac{V_1^2}{2g}=\frac{p_2}{\gamma}+z_2+\frac{V_2^2}{2g}$$

2. The pressure at the outlet and the tank surface are the same (atmospheric), so $p_1 = p_2$. The velocity at the tank surface is much less than in the drain port so $V_2^2 \gg V_1^2$. Solution for $V_2$:

$$z_1 - z_2 = \frac{V_2^2}{2g}$$

$$V_2 = \sqrt{2g(z_1 - z_2)}$$

3. Velocity calculation:

$$V_2 = \sqrt{2 \times 9.81 \text{ m/s}^2 \times 10 \text{ m}}$$

$$= \boxed{14 \text{ m/s}}$$

*Review*

1. Notice that the answer is independent of liquid properties. This would be true for all liquids so long as viscous effects are unimportant. Also note that the same velocity would be calculated for an object dropped from the same elevation as the liquid in the tank.
2. Selection of point 1 is not critical; it can be taken at any point on liquid surface.
3. The assumption of the small velocity at the liquid surface is generally valid. It will be shown in Chapter 5 that the ratio of the velocity at the liquid surface to the drain port velocity is

$$\frac{V_1}{V_2} = \frac{A_2}{A_1}$$

where $A_2$ is the cross-sectional area of the drain port and $A_1$ is the cross-sectional area of the tank. For example with $A_2/A_1 = 0.1$, $V_1^2 = 0.01\,V_2^2$.

EXAMPLE 4.10 EVALUATION OF ROTATION OF VELOCITY FIELD

The vector $\mathbf{V} = 10x\mathbf{i} - 10y\mathbf{j}$ represents a two-dimensional velocity field. Is the flow irrotational?

*Problem Definition*

**Situation:** Velocity field given.

**Find:** If flow is irrotational.

*Plan*

Flow is two-dimensional, so $w = 0$ and $\frac{\partial}{\partial z} = 0$. Use Eq. (4.30a) to evaluate rotationality.

*Solution*

Velocity components and derivatives

$$u = 10x \qquad \frac{\partial u}{\partial y} = 0$$

$$v = -10y \qquad \frac{\partial v}{\partial x} = 0$$

Thus flow is irrotational.

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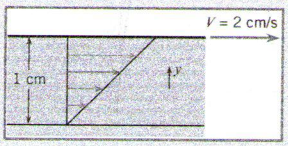
EXAMPLE 4.11 ROTATION OF A FLUID ELEMENT

A fluid exists between stationary and moving parallel flat plates, and the velocity is linear as shown. The distance between the plates is 1 cm, and the upper plate moves at 2 cm/s. Find the amount of rotation that the fluid element located at 0.5 cm will undergo after it has traveled a distance of 1 cm.

**Sketch:**




*Problem Definition*

**Situation:** Flow between moving, parallel, flat plates.

**Find:** Rotation of fluid element at midpoint after traveling 1 cm.

**Assumptions:** Planar flow ($w = 0$ and $\frac{\partial}{\partial z} = 0$).

*Plan*

1. Use Eq. (4.28a) to evaluate rotational rate with $v = 0$.
2. Find time for element to travel 1 cm.
3. Calculate amount of rotation.

*Solution*

1. Velocity distribution

$$u = 0.02 \text{ m/s} \times \frac{y}{0.01 \text{ m}} = 2y \text{ (1/s)}$$

Rotational rate

$$\Omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = -1 \text{ rad/s}$$

2. Time to travel 1 cm:

$$u = 2 \text{ (1/s)} \times 0.005 \text{ m} = 0.01 \text{ m/s}$$

$$\Delta t = \frac{\Delta x}{u} = \frac{0.01 \text{ m}}{0.01 \text{ m/s}} = 1s$$

3. Amount of rotation

$$\Delta\theta = \Omega_z \times \Delta t = -1 \times 1 = -1 \text{ rad}$$

*Review*

Note that the rotation is negative (in clockwise direction).

EXAMPLE 4.12 VELOCITY AND PRESSURE DISTRIBUTION IN A FREE VORTEX

A free vortex in air rotates in a horizontal plane and has a velocity of 40 m/s at a radius of 4 km from the vortex center. Find the velocity at 10 km from the center and the pressure difference between the two locations. The air density is 1.2 kg/m$^3$.

*Problem Definition*

**Situation:** Free vortex in horizontal plane.

**Find:**

1. Velocity (in m/s) 10 km from center.
2. Pressure difference (Pa) between two radii.

**Assumptions:** Flow is incompressible and steady.

**Properties:** $\rho = 1.2 \text{ kg/m}^3$.

*Plan*

1. Apply Eq. (4.34) to calculate velocity.
2. Since flow is irrotational, use the Bernoulli equation, Eq. (4.40), for pressure difference.

*Solution*

1. Velocity distribution

$$V = \frac{C}{r}$$

$$\frac{V_{10\text{km}}}{V_{4\text{km}}} = \frac{r_{4\text{km}}}{r_{10\text{km}}} = 0.4$$

$$V_{10\text{km}} = 0.4 \times 40$$

$$= \boxed{16 \text{ m/s}}$$

2. The Bernoulli equation for a horizontal plane

$$p_{4\text{km}} + \rho\frac{V_{4\text{km}}^2}{2} = p_{10\text{km}} + \rho\frac{V_{10\text{km}}^2}{2}$$

$$p_{10\text{km}} - p_{4\text{km}} = \frac{\rho}{2}(V_{4\text{km}}^2 - V_{10\text{km}}^2)$$

$$= \frac{1.2 \text{ kg/m}^3}{2}(40^2 - 16^2) \text{ (m/s)}^2$$

$$= \boxed{806 \text{ Pa}}$$

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**EXAMPLE 4.13 PRESSURE DIFFERENCE IN TORNADO**

Assume that a tornado is modeled as the combination of a forced and a free vortex. The maximum wind speed in the tornado is 150 mph. What is the pressure difference, in inches of mercury, between the center and the outer edge of the tornado? The density of the air is 0.075 lbm/ft$^3$.

*Problem Definition*

**Situation:** Tornado with 150 mph winds.

**Find:** Pressure difference (inches Hg) between center and edge.

**Assumptions:** Tornado modeled as forced and free vortex.

**Properties:** $\rho = 0.075$ lbm/ft$^3$.

*Plan*

1. Use Eq. (4.49) to calculate pressure difference.
2. Convert result to inches Hg.

*Solution*

1. Convert velocity to ft/s:

$$150\frac{\text{mi}}{\text{hr}} \times \frac{5280\text{ ft}}{\text{mi}} \times \frac{\text{hr}}{3600\text{ s}} = 220\frac{\text{ft}}{\text{s}}$$

Convert density to slug/ft$^3$:

$$0.075\frac{\text{lbm}}{\text{ft}^3} \times \frac{\text{slug}}{32.2\text{ lbm}} = 0.00233\frac{\text{slug}}{\text{ft}^3}$$

Pressure difference

$$p_1 - p_0 = -\rho(V_{\max})^2$$

$$p_1 - p_0 = -0.00233\frac{\text{slugs}}{\text{ft}^3} \times 220^2\frac{\text{ft}^2}{\text{s}^2} = -112.8\text{ psf}$$

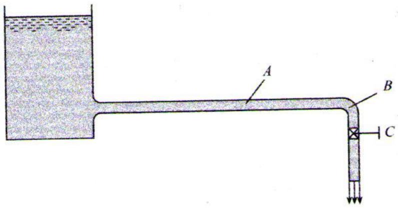
2. Convert to inches Hg:

$$p_1 - p_0 = -112.8\text{ psf} \times \frac{29.92\text{ in Hg}}{2116\text{ psf}} = -1.59\text{ in Hg}$$

**4.4** In the system in the figure, the valve at $C$ is gradually opened in such a way that a constant rate of increase in discharge is produced. How would you classify the flow at $B$ while the valve is being opened? How would you classify the flow at $A$?

B ⇒ nonuniform, unsteady

A ⇒ uniform & unsteady



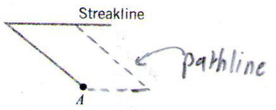

PROBLEM 4.4

**4.7** Consider flow in a straight conduit. The conduit is circular in cross section. Part of the conduit has a constant diameter, and part has a diameter that changes with distance. Then, relative to flow in that conduit, correctly match the items in column A with those in column B.

| A | B | |
|---|---|---|
| Steady flow | $\partial V_s/\partial s = 0$ | uniform |
| Unsteady flow | $\partial V_s/\partial s \neq 0$ | nonuniform |
| Uniform flow | $\partial V_s/\partial t = 0$ | steady |
| Nonuniform flow | $\partial V_s/\partial t \neq 0$ | unsteady |

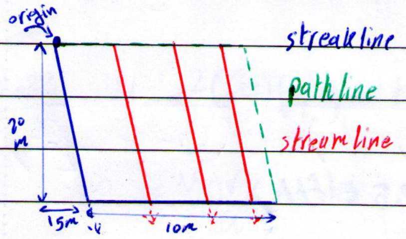
Subject Date No.

**4.11** At time $t = 0$, dye was injected at point $A$ in a flow field of a liquid. When the dye had been injected for 4 s, a pathline for a particle of dye that was emitted at the 4 s instant was started. The streakline at the end of 10 s is shown below. Assume that the speed (but not the velocity) of flow is the same throughout the 10 s period. Draw the pathline of the particle that was emitted at $t = 4$ s. Make your own assumptions for any missing information.

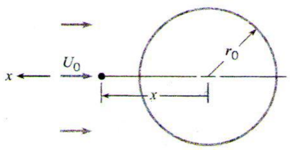



PROBLEM 4.11

**4.12** For a given hypothetical flow, the velocity from time $t = 0$ to $t = 5$ s was $u = 2$ m/s, $v = 0$. Then, from time $t = 5$ s to $t = 10$ s, the velocity was $u = +3$ m/s, $v = -4$ m/s. A dye streak was started at a point in the flow field at time $t = 0$, and the path of a particle in the fluid was also traced from that same point starting at the same time. Draw to scale the streakline, pathline of the particle, and streamlines at time $t = 10$ s.




**4.18** Tests on a sphere are conducted in a wind tunnel at an air speed of $U_0$. The velocity of flow toward the sphere along the longitudinal axis is found to be $u = -U_0(1 - r_0^3/x^3)$, where $r_0$ is the radius of the sphere and $x$ the distance from its center. Determine the acceleration of an air particle on the $x$-axis upstream of the sphere in terms of $x$, $r_0$, and $U_0$.




PROBLEM 4.18

$$a_x = u\frac{\partial u}{\partial x} + \frac{\partial u}{\partial t}$$

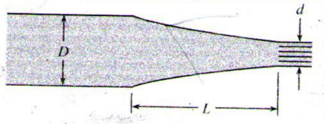
$$= \left[U_0\left(1 - \frac{r_0^3}{x^3}\right)\right]\left[\frac{\partial u}{\partial x}\left(-U_0\left(1 - \frac{r_0^3}{x^3}\right)\right)\right] + \left[\frac{\partial}{\partial t}\left(-U_0\left(1 - \frac{r_0^3}{x^3}\right)\right)\right]$$

$$= U_0^2\left(1 - \frac{r_0^3}{x^3}\right)\left(\frac{-3r_0^3}{x^4}\right) + 0$$

$$a_x = -\left(\frac{3U_0^2 r_0^3}{x^4}\right)\left(1 - \frac{r_0^3}{x^3}\right)$$

Subject Date No.

**4.20** The nozzle in the figure is shaped such that the velocity of flow varies linearly from the base of the nozzle to its tip. Assuming quasi-one-dimensional flow, what is the convective acceleration midway between the base and the tip if the velocity is 1 ft/s at the base and 4 ft/s at the tip? Nozzle length is 18 inches.




PROBLEMS 4.20, 4.21

Velocity gradient $\frac{\partial v}{\partial s} = \frac{(V_{tip} - V_{base})}{L} = \frac{(4-1)}{1,5} = 2\ s^{-1}$

acceleration at midpoint $V = \frac{(1+4)}{2} = 2,5\ ft/s$

$a_c = V\frac{dv}{\partial s} = 2,5 * 2 = 5\ ft/s^2$

local acceleration $a_t = 0$

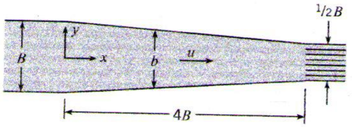
**4.21** In Prob. 4.20 the velocity varies linearly with time throughout the nozzle. The velocity at the base is $1t$ (ft/s) and at the tip is $4t$ (ft/s). What is the local acceleration midway along the nozzle when $t = 2$ s?

$a_t = \frac{\partial V}{\partial t} \Rightarrow V = \frac{(t+4t)}{2} = 2,5\,t\ (ft/s)$

$a_t = \frac{\partial}{\partial t}(2,5t) \Rightarrow a_t = 2,5\ ft/s^2$

Subject Date No.

**4.22** Liquid flows through this two-dimensional slot with a velocity of $V = 2(q_0/b)(t/t_0)$, where $q_0$ and $t_0$ are reference values. What will be the local acceleration at $x = 2B$ and $y = 0$ in terms of $B$, $t$, $t_0$, and $q_0$?




PROBLEMS 4.22, 4.23

**4.23** What will be the convective acceleration for the conditions of Prob. 4.22?

4.22) $V = 2\left(\frac{q_0}{b}\right)\left(\frac{t}{t_0}\right)$, Find $a_l$ midway in nozzle:-

$$b = \frac{B}{2} \Rightarrow V = \left(\frac{4q_0}{B}\right)\left(\frac{t}{t_0}\right)$$

$$a_l = \frac{\partial V}{\partial t} = \frac{4q_0}{Bt_0}$$

4.23) $V = 2\left(\frac{q_0}{b}\right)\left(\frac{t}{t_0}\right)$, Find $a_c$ midway in nozzle:-

$$a_c = V\frac{\partial V}{\partial x} \Rightarrow \boxed{b = B - \frac{x}{8}} \rightarrow \text{width varies}$$

$$V = \left(\frac{q_0}{t_0}\right)\frac{2t}{\left(B - \frac{x}{8}\right)} \Rightarrow \frac{q_0}{t_0}\,2t\left(B - \frac{x}{8}\right)^{-1}$$

$$\frac{\partial V}{\partial x} = \left(\frac{q_0}{t_0}\right)2t\;\frac{1}{8}\;\left(B - \frac{x}{8}\right)^{-2}$$

$$a_c = V\frac{\partial V}{\partial x} = \frac{V\left(\frac{q_0}{t_0}\right)^2 4t^2\left(\frac{1}{8}\right)}{\left(B - \frac{1}{8}x\right)^{-3}}$$

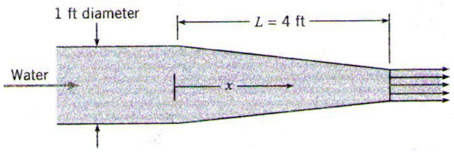
at $x = 2B$

$$a_c = \frac{\left(\frac{1}{2}\right)\left(\frac{q_0}{t_0}\right)^2 t^2}{\left(\left(\frac{3}{4}\right)B\right)^3}$$

**4.24** The velocity of water flow in the nozzle shown is given by the following expression:

$$V = 2t/(1 - 0.5x/L)^2,$$

where $V$ = velocity in feet per second, $t$ = time in seconds, $x$ = distance along the nozzle, and $L$ = length of nozzle = 4 ft. When $x = 0.5L$ and $t = 3$ s, what is the local acceleration along the centerline? What is the convective acceleration? Assume quasi-one-dimensional flow prevails.




PROBLEM 4.24

$$a_l = \frac{\partial v}{\partial t} = \frac{\partial}{\partial t}\left(\frac{2t}{(1-\frac{0.5x}{L})^2}\right)$$

$$= \frac{2}{(1-\frac{0.5x}{L})^2}$$

$$= \frac{2}{(1-\frac{0.5 * 0.5L}{L})^2}$$

$$\boxed{a_L = 3.56\ ft/s^2}$$

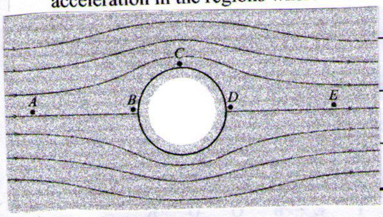
$$a_c = v\frac{\partial v}{\partial x} = \left(\frac{2t}{(1-\frac{0.5x}{L})^2}\right)\frac{\partial}{\partial x}\left(\frac{2t}{(1-\frac{0.5x}{L})^2}\right)$$

$$= \frac{4t^2}{((1-\frac{0.5x}{L})^5\ L)} = \frac{4(3)^2}{((1-\frac{0.5*0.5L}{L})^5 * 4)}$$

$$\boxed{a_c = 37.9\ ft/s}$$

**4.16** Figure 4.24 on p. 110 shows the flow pattern for flow past a circular cylinder. Assume that the approach velocity at $A$ is constant (does not vary with time).

a. Is the flow past the cylinder steady or unsteady?
b. Is this a case of one-dimensional, two-dimensional, or three-dimensional flow?
c. Are there any regions of the flow where local acceleration is present? If so, show where they are and show vectors representing the local acceleration in the regions where it occurs.
d. Are there any regions of flow where convective acceleration is present? If so, show vectors representing the convective acceleration in the regions where it occurs.




a ⇒ steady

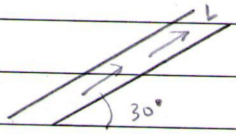
b ⇒ two dimensional

c ⇒ No

d ⇒ yes, convective acceleration is present at all local where the stream lines curve. Also $a_c$ is present at each where a fluid particles changes speed as it moves along the streamline

Subject Date No.

**4.27** A pipe slopes upward in the direction of liquid flow at an angle of 30° with the horizontal. What is the pressure gradient in the flow direction along the pipe in terms of the specific weight of the liquid if the liquid is decelerating (accelerating opposite to flow direction) at a rate of $0.3g$?




Apply Euler's equation

$$\frac{\partial}{\partial L}(p+\gamma z) = -\rho a_L$$

$$\frac{\partial p}{\partial L} + \gamma \frac{\partial z}{\partial L} = -\rho a_L$$

$$\frac{\partial p}{\partial L} = -\rho a_L - \gamma \frac{\partial z}{\partial L}$$

$$= -\left(\frac{\gamma}{g}\right) * (-0{,}30g) - \gamma \sin 30$$

$$= \gamma(0{,}3 - 0{,}5)$$

$$\boxed{\frac{\partial p}{\partial L} = -0{,}2\gamma}$$

**4.28** What pressure gradient is required to accelerate kerosene ($S = 0.81$) vertically upward in a vertical pipe at a rate of $0.3\,g$?

Apply Euler's equation in the Z direction

$$\frac{\partial(p+\gamma z)}{\partial z} = -\rho a_z = -\left(\frac{\gamma}{g}\right) * 0{,}3g$$

$$\frac{\partial p}{\partial z} + \gamma = -0{,}3\,g$$

$$\frac{\partial p}{\partial z} = \gamma(-1 - 0{,}3)$$

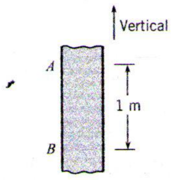
$$= 0{,}8 * 62{,}4 * (-1{,}3)$$

$$\frac{\partial p}{\partial z} = -64{,}896 \ lbf/ft^3$$

NOTEBOOK

Subject Date No.

**4.29** The hypothetical liquid in the tube shown in the figure has zero viscosity and a specific weight of 10 $kN/m^3$. If $p_B - p_A$ is equal to 12 kPa, one can conclude that the liquid in the tube is being accelerated (a) upward, (b) downward, or (c) neither: acceleration = 0.




PROBLEM 4.29

$$\rho a_l = -\frac{\partial}{\partial l}(p + \gamma z)$$

$$a_l = \left(\frac{1}{\rho}\right)\left(-\frac{\partial p}{\partial l} - \gamma \frac{\partial z}{\partial l}\right)$$

let $l$ be positive upward

Then $\frac{\partial z}{\partial l} = 1$

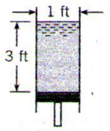
and $\frac{\partial p}{\partial l} = \frac{(p_A - p_B)}{1} = -12{,}000$

$$a_l = \left(\frac{g}{\gamma}\right)(12{,}000 - \gamma)$$

$$a_l = g\left(\frac{12000}{\gamma} - 1\right) = g(1{,}2 - 1{,}0) = 0{,}2\ m/s^2$$

$\boxed{a}$

**4.30** If the piston and water ($\rho = 62.4\ lbm/ft^3$) are accelerated upward at a rate of 0.5$g$, what will be the pressure at a depth of 2 ft in the water column?




PROBLEM 4.30

$$\rho a_l = -\frac{\partial}{\partial l}(p + \gamma z)$$

let $l$ positive upward

$$\rho(0{,}5g) = -\frac{\partial p}{\partial l} - \gamma\frac{\partial z}{\partial l}$$

$$\left(\frac{\gamma}{g}\right)(0{,}5g) = -\frac{\partial p}{\partial l} - \gamma(1)$$

$$\left(\frac{\partial p}{\partial l}\right) = -\gamma(0{,}5 + 1) = -1{,}5\gamma$$

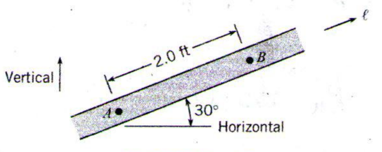
Thus the $p$ decreases upward at rate $1{,}5\gamma$. At depth 2ft g-

$$p_2 = (1{,}5\gamma)(2) = 3\gamma$$

$$= 3 * 62{,}4 = 187{,}2\ psfg$$

Subject Date No.

**4.35** A liquid with a specific weight of 100 lbf/ft³ is in the conduit. This is a special kind of liquid that has zero viscosity. The pressures at points $A$ and $B$ are 170 psf and 100 psf, respectively. Which one (or more) of the following conclusions can one draw with certainty? (a) The velocity is in the positive $\ell$ direction. (b) The velocity is in the negative $\ell$ direction. (c) The acceleration is in the positive $\ell$ direction. (d) The acceleration is in the negative $\ell$ direction.




PROBLEM 4.35

$$-\frac{\partial}{\partial l}(p+\gamma z) = \rho a_l$$

$$-\frac{\partial p}{\partial l} - \gamma\frac{\partial z}{\partial l} = \rho a_l$$

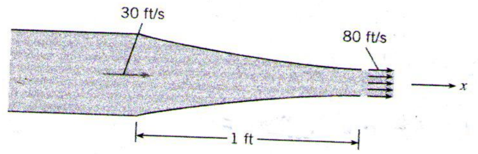
where $\frac{\partial p}{\partial l} = \frac{(p_B - p_A)}{L} = \frac{(100-170)}{2} = -35$ lbf/ft³

$$\frac{\partial z}{\partial l} = \sin 30 = 0.5$$

$$a_l = \left(\frac{1}{\rho}\right)(35 - 100 * 0.5) = \left(\frac{1}{\rho} * -15\right) \text{ lbf/ft}^3$$

$\boxed{d}$

**4.36** If the velocity varies linearly with distance through this water nozzle, what is the pressure gradient, $dp/dx$, halfway through the nozzle? ($\rho = 62.4$ lbm/ft³).




PROBLEM 4.36

$$\frac{d}{dx}(p+\gamma z) = -\rho a_x$$

but $z$ constant

$$\frac{dp}{dx} = -\rho a_x$$

$$a_x = a_{convective} = V\frac{dV}{dx}$$

$$\frac{dV}{dx} = \frac{(80-30)}{1} = 50\, s^{-1}$$

$$V_{mid} = \frac{(80+30)}{2} = 55 \text{ ft/s}$$

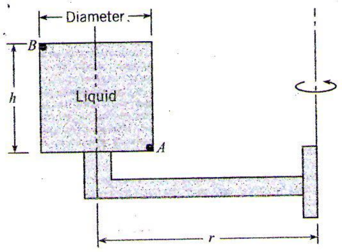
$$a_x = 55 * 50 = 2750 \text{ ft/s}^2$$

$$\frac{dp}{dx} = -1.94 * 2750 = -5335 \text{ psf/ft}$$

Poletechnic Lecture Note

Subject Date No.

**4.42** A tank of liquid (S = 0.80) that is 1 ft in diameter and 1.0 ft high ($h$ = 1.0 ft) is rigidly fixed (as shown) to a rotating arm having a 2 ft radius. The arm rotates such that the speed at point $A$ is 20 ft/s. If the pressure at $A$ is 25 psf, what is the pressure at $B$?




PROBLEM 4.42

$$p_A + \gamma z_A + \frac{\rho r_A^2 \omega^2}{2}$$

$$= p_B + \gamma z_B + \frac{\rho r_B^2 \omega^2}{2}$$

$$\Rightarrow p_B = p_A + \left(\frac{\rho}{2}\right)\omega^2 (r_B^2 - r_A^2)$$

$$+ \gamma (z_A - z_B)$$

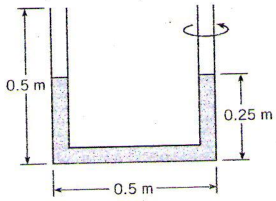
where $\omega = \frac{V_A}{r_A} = \frac{20}{1{,}5} = 13{,}333$ rad/s and $\rho = 0{,}8 \times 1{,}94$

$$p_B = 25 + \left(\frac{1{,}94 \times 0{,}8}{2}\right)(13{,}33^2)(2{,}5^2 - 1{,}5^2)$$

$$+ 62{,}4 \times (0{,}8 \times -1) = 25 + 551{,}5 - 49{,}9$$

$$\boxed{p_B = 526{,}6 \text{ psf}}$$

**4.45** A U-tube is rotated about one leg, as shown. Before being rotated the liquid in the tube fills 0.25 m of each leg. The length of the base of the U-tube is 0.5 m, and each leg is 0.5 m long. What would be the maximum rotation rate (in rad/s) to ensure that no liquid is expelled from the outer leg?




PROBLEM 4.45

$$p_1 + \gamma z_1 - \frac{\rho r_1^2 \omega^2}{2} = p_2 + \gamma z_2 - \frac{\rho r_2^2 \omega^2}{2}$$

where $p_1 = p_2 = 0{,}5$ m and $z_2 = 0$

$$\gamma \times 0{,}5 - \left(\frac{\gamma}{g}\right) \times \frac{0{,}5^2 \omega^2}{2} = 0$$

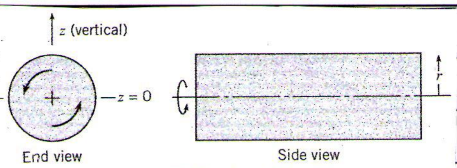
$$\omega^2 = 4g$$

$$\omega = 2\sqrt{g}$$

$$\boxed{\omega = 6{,}26 \text{ rad/s}}$$

Subject Date No.

**4.54** A closed cylindrical tank of water ($\rho = 1000\ kg/m^3$) is rotated about its horizontal axis as shown. The water inside the tank rotates with the tank ($V = r\omega$). Derive an equation for $dp/dz$ along a vertical-radial line through the center of rotation. What is $dp/dz$ along this line for $z = -1$ m, $z = 0$, and $z = +1$ m when $\omega = 5$ rad/s? Here $z = 0$ at the axis.




PROBLEMS 4.54, 4.55, 4.56

$$\frac{\partial p}{\partial r} + \gamma \frac{\partial z}{\partial r} = -\rho r \omega^2$$

$$\frac{\partial p}{\partial z} = -\gamma - \rho r \omega^2$$

When $z = -1$ m $\Rightarrow \frac{\partial p}{\partial z} = -\gamma - \rho \omega^2 = -\gamma\left(1 + \frac{\omega^2}{g}\right)$

$$= -9810\left(1 + \frac{25}{9{,}81}\right)$$

$$\frac{\partial p}{\partial z} = -34{,}8\ kPa/m$$

when $z = +1$ m $\Rightarrow$

$$\partial p / \partial z = -\gamma + \rho \omega^2$$

$$= -\gamma(1 + \omega^2/g)$$

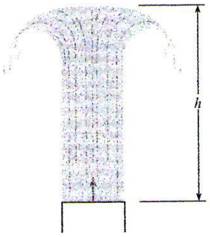
$$= -9810\left(1 - \frac{25}{9{,}81}\right) = 15{,}19\ kPa/m$$

At $z = 0$ m $\Rightarrow$

$$\frac{\partial p}{\partial z} = -\gamma = -9810\ Npa/m$$

Subject Date No.

**4.59** A water jet issues vertically from a nozzle, as shown. The water velocity as it exits the nozzle is 20 ft/s. Calculate how high $h$ the jet will rise. (*Hint:* Apply the Bernoulli equation along the centerline.)




PROBLEM 4.59

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\gamma}+\frac{V_2^2}{2g}+z_2$$

where $P_1 = P_2 = 0$ gage

$V_1 = 20$ ft/s

$V_2 = 0$

$$0+\frac{20^2}{2g}+z_1 = 0+0+z_2$$

$$z_2 - z_1 = h = \frac{400}{64.4} = 6.21$$

$$\boxed{h = 6.21\ ft}$$

**4.63** A Pitot-static tube is mounted on an airplane to measure airspeed. At an altitude of 10,000 ft, where the temperature is 23°F and the pressure is 10 psia, a pressure difference corresponding to 10 in of water is measured. What is the airspeed?

Pilot tube equation $\Rightarrow$ $V = \sqrt{2\,\Delta P_z/\rho}$

$\Delta P_z = \gamma_{H_2O}\, h_{H_2O}$

$= 62.4 * (10/12) = 52$ psf

Ideal gas law $\Rightarrow$ $\rho = P/RT$

$= (10*144)/(1716*483)$

$= 0.00174$ slugs/ft$^3$

$$V = \sqrt{\frac{2*52}{0.00174}} = 244\ ft/s$$

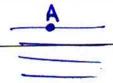
## Ch5:- control volume approach and continuity equation

Subject Date No.

* Lagrangain view point

$\vec{r} = x_i + y_j + z_k$

$\vec{V} = u_i + v_j + w_k$

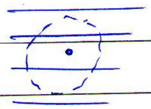



* Eulerian view point

$u = f(x, y, z, t)$

$v = f(x, y, z, t)$

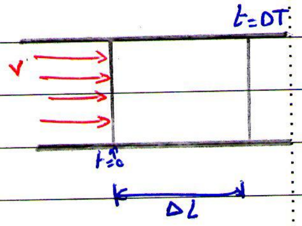
$w = f(x, y, z, t)$



تركيز على volume محدد

* Volume flow rate :-

$\Delta L = V \Delta T$ الزمن الذي تقطعه




$\dot{\forall} = \frac{\forall}{t} \quad [m^3/s]$

Volume flow rate (discharge) (Q)

$\forall = \Delta L A$

$\forall = V \Delta T A$

$\frac{\forall}{\Delta T} = VA$

$\dot{\forall} = Q = VA$

$\underset{\text{scallar}}{Q} = V \cdot A \Rightarrow$ uniform V ازاكانت

$Q = \int v \cdot dA$

discharge: دخول او خروج flow من برميل

$\underset{\text{(mean)}}{\bar{V}_{avg.}} = \frac{Q}{A} \qquad \bar{V} = \frac{1}{A} \int v \, dA$

السرعة المتوسطة

NOTEBOOK

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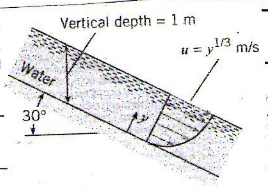
**5.19** If the velocity in the channel of Prob. 5.18 is given as $u = 10[\exp(y) - 1]$ m/s and the channel width is 2 m, what is the discharge in the channel and what is the mean velocity?



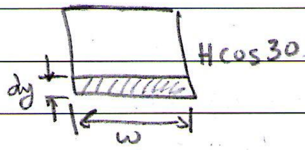

PROBLEM 5.18

width = 2 m

$Q = ??$

$\bar{V} = ??$

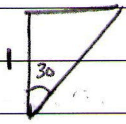
$u = 10\,(e^{y} - 1)$ m/s




* $u$ تتغير مع $y$ وليست ثابتة لذلك نستخدم $\int$

$Q = \int v\, dA$

$$\boxed{\begin{aligned} dA &= w\, dy \\ dA &= 2\, dy \end{aligned}}$$




$y_{max} = h\cos 30$

$= 1 \times \cos 30$

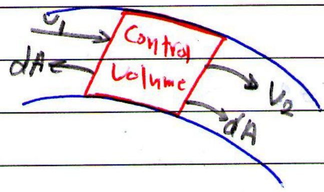
$= 0{,}866$

$$Q = \int_0^{0{,}866} 10\,(e^{y} - 1) \times 2\, dy = 20\,(e^{y} - y)\Big|_0^{0{,}866}$$

$$Q = 20 \times \left[ (e^{0{,}866} - 0{,}866) - (e^{0}) \right] = 10{,}227 \ m^2/s$$

$$\bar{V} = \frac{Q}{A} = \frac{10{,}227}{(2 \times 0{,}866)} = 5{,}905 \ m/s$$

* control volume approach:-




* نأخذ حجم ثابت من Flow وهذا ∀ في surface
وبالتالي الخطوط ▭ تمثل control surface
، عادة يتم التركيز على الـ control volume لدراسة
ما يحدث داخله
* dA دائماً عمودي على السطح

أي خاصية ⇐ $B$ = extensive properties
تعتمد على m

$b = \frac{B}{m} \Longrightarrow$ specific property (intensive)

$B = mb$

* أي خاصية داخل C.∀ نقسم إلى ← ① ما يحدث داخل C.∀
② على حدود C.∀ ← حول C.S

* control volume:-

$$B_{\text{inside C.∀}} = mb = m_{c.∀}\, b = \rho V_{c.v}\, b = \int \rho\, b\, d\forall$$

إذا كانت ثابتة تخرج من التكامل

$$\dot{B} = \frac{B}{\Delta t} = \frac{d}{dt}\int \rho\, b\, d\forall$$

تعتمد على مقدار الخاصية داخل C.∀ مع الزمن

$$\text{Power} = \frac{U}{t}$$

إذا كانت هنا الخاصية U أو $\dot{B}$ هي عبارة عن الـ Power

* control surface:~ * الخاصية تدخل للنظام عبر الكتلة التي دخلت فإذا الكتلة التي دخلت لها (U) بتدخل معها (U) وإذا الكتلة خرجت تخرج معها (V) ((الكتلة الداخلة والخارجة تدخل وتخرج معها الخاصية))

$$\text{Net mass flow rate} = \dot{m}_{out} - \dot{m}_{in}$$

صافي التدفق الكلي

$$\dot{m} = \rho Q \qquad [kg/s] = \frac{kg}{m^3} * \frac{m^3}{s}$$

$$= \rho \bar{V} A$$

معدل الكتلة التي تعبر خلال المساحة

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$$\dot{m}_{out} = \rho V_2 A_2 - \rho V_1 A_1 = \sum_{c.s} \rho V A \Rightarrow \text{uniform}$$

For steady = 0

$$= \int \rho V dA \Rightarrow cs$$ اذا كانت لا تتغير

** $\dot{B}$ through control surface:-

$$\dot{B} = b\dot{m}$$

$$(\dot{B})_{c.s} = \sum b \rho V \cdot A \Rightarrow \text{uniform}$$

$$= \int b \rho V \cdot dA$$ اذا كانت متغيرة

Renolnd ~~transport~~ transport

$$\boxed{\frac{dB}{dt} = \frac{d}{dt}\int_{c.\forall} \rho b \, d\forall + \int_{c.s} b \rho V dA}$$

وإذا كانت المساحة غير متغيرة A

$$= \frac{d}{dt}\int_{c.\forall} \rho b \, d\forall + \int_{c.s} b \rho V A$$

For steady = 0

$B = m$

$b = 1$

$$\frac{dm}{dt} = \frac{d}{dt}\int_{c.\forall} \rho \, d\forall + \int_{c.s} \rho v dA$$

= 0 لأن m لا تتغير t

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* continuity equation :-

if $B = m \Rightarrow b = 1$

$$\frac{d}{dt}(m)_{sys} = \frac{d}{dt}\int_{c.\forall} \rho \, d\forall + \int_{c.s} \rho V dA = 0$$

$$\frac{d}{dt}\int_{c.\forall} \rho \, d\forall = -\int_{c.s} \rho V d\forall = -\sum_{c.s} \rho VA = \left((\rho VA)_o - (\rho VA)_i\right)$$ دخل

$$\frac{d}{dt}\int_{c.\forall} \rho \, d\forall = \left((\rho VA)_i - (\rho VA)_e\right)$$

* For steady :- لايوجد تغير m مع t داخل C.∀

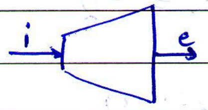
$$\sum_{c.s} \rho VA = 0 \Rightarrow \sum \dot{m}_i = \sum \dot{m}_e$$

$$\sum(\rho VA)_i = \sum(\rho VA)_e \quad [kg/s]$$

$$\sum(VA)_i = \sum(VA)_e \quad [m^3/s]$$

$$\sum Q_i = \sum Q_e$$

** Flow in pipe :-




$\dot{m}_i = \dot{m}_e$

Flow incompressible $\Rightarrow$ $\rho$ constant

$$Q_i = Q_e$$

$$\boxed{V_i A_i = V_e A_e}$$

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* differential form of continuity equation :-

$$\frac{\partial}{\partial x}\rho u + \frac{\partial}{\partial y}\rho v + \frac{\partial}{\partial z}\rho w + \frac{\partial}{\partial t}\rho = 0$$

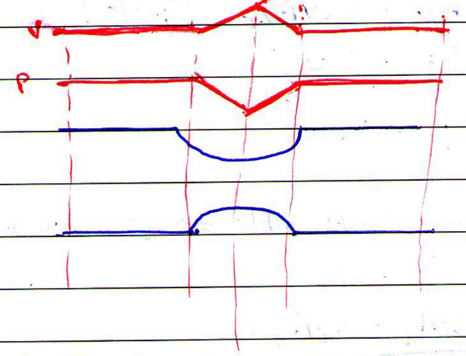
$\Rightarrow$ For $\rho$ constant

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial \rho}{\partial t} = 0$$

$\Rightarrow$ For incompressible + steady

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

** Covitation :- ظاهرة التكهف

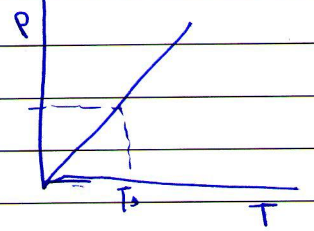



$$\frac{P}{\gamma} + \frac{V^2}{2g} = C$$

$$Z_1 = Z_2$$

* saturation pressure :- ضغط الاشباع

تبخر جزء من Fluid نتيجة لانخفاض
الضغط داخل النظام



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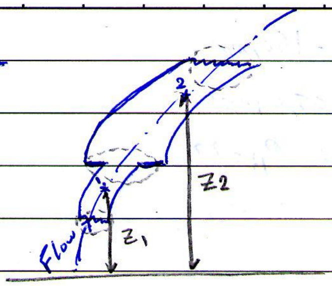
**




$D_1 = 25mm$

$D_2 = 50 mm$

$V_1 = 3 m/s$

$P_1$ gauge $= 345$ Kpa

$Z_2 - Z_1 = 2m$

$P_2 = ??$

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g}$$

$$P_2 = P_1 + \gamma(Z_1 - Z_2) + \left(\frac{V_1^2 - V_2^2}{2g}\right) * \gamma$$

$m_i = m_e$

$Q_i = Q_e$

$V_1 A_1 = V_2 A_2$

$$V_2 = \frac{A_1}{A_2} V_1 = \frac{\frac{\pi}{4}(0{,}025)^2}{\frac{\pi}{4}(0{,}05)^2} * V_1$$

$V_2 = 0{,}75$ m/s

$$P_2 = 345 + 9{,}81(-2) + 9{,}81\left(\frac{3^2 - 0{,}75^2}{2 * 9{,}81}\right)$$

$= 329{,}6$ kpa (gauge)

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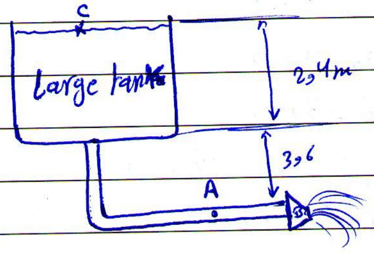
ex:-




$D_A = 150\,mm$

$D_B = 50\,mm$

Find $P_A = ??$

* يتم افتراض V عند C = 0

لأنه خزان كبير

$$\frac{\cancel{P_c}}{\cancel{\gamma}} + Z_c + \frac{V_c^2}{2g} = \frac{\cancel{P_B}}{\cancel{\gamma}} + Z_B + \frac{V_B^2}{2g}$$

$V_c \approx 0$

$P_B = P_c = 0$ gauge

$$V_B = \sqrt{2g\,(Z_c - Z_B)}$$

$V = 0$

سقوط الجسم الحر

$V = \sqrt{2gh}$

$$= \sqrt{2 * 9{,}81 * 6}$$

$$= 10{,}84\ m/s$$

$$V_A A_A = V_B A_B \Rightarrow V_A = \frac{V_B A_B}{A_A} = 10{,}84 * \frac{\frac{\pi}{4}(0{,}05)^2}{\frac{\pi}{4}(0{,}15)^2}$$

$$= 1{,}204\ m/s$$

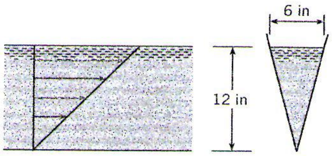
$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + \cancel{Z_A} = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + \cancel{Z_B}$$

$$P_A = \gamma\left(\frac{V_B^2 - V_A^2}{2g}\right) = 9{,}81\left(\frac{10{,}84^2 - 1{,}204^2}{2 * 9{,}81}\right)$$

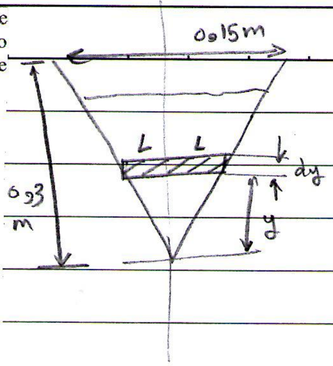
$$= 58{,}3\ KPa$$

NOTEBOOK

**5.23** The hypothetical water velocity in a V-shaped channel (see the accompanying figure) varies linearly with depth from zero at the bottom to maximum at the water surface. Determine the discharge if the maximum velocity is 6 ft/s.




PROBLEM 5.23




$V_{max} = 2\,m/s$

$Q = ??$

$$Q = VA = \int V \cdot dA$$

$$V = a + cy$$

$$V(0) = 0 \Rightarrow \boxed{a = 0}$$

$$\boxed{V(0,3) = 2}$$

$$2 = c * 0,3 \Rightarrow \boxed{c = 6,67}$$

$$\boxed{V = 6,67y}$$

$$dA = 2L\,dy$$

$$\frac{L}{y} = \frac{0,075}{0,3} = \frac{1}{4}$$

$$4L = y$$

$$\boxed{2L = \frac{y}{2}}$$

$$dA = \frac{y}{2}\,dy$$

$$Q = \int_0^{0,3} (6,67y)\left(\frac{y}{2}dy\right) = \frac{6,67}{2}\int_0^{0,3} y^2\,dy$$

$$= \frac{6,67}{6}\, y^3 \Big|_0^{0,3}$$

$$= 0,03\ m^3/s$$

$$= 30\ L/s$$

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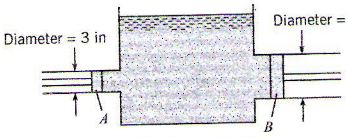
**5.44** Both pistons are moving to the left, but piston $A$ has a speed twice as great as that of piston $B$. Then the water level in the tank is (a) rising, (b) not moving up or down, or (c) falling?




PROBLEM 5.44

$V_A = 2V_B$

$$-\frac{d}{dt}\int \rho \, d\forall = \sum_{c.s} \dot{m} = \sum \dot{m}_e - \sum \dot{m}_i$$

$$\frac{d}{dt}\int \rho \, d\forall = \sum \dot{m}_i - \sum \dot{m}_e$$

$$= \cancel{\rho} V_B A_B - \cancel{\rho} V_A A_A$$

$$\frac{d}{dt}\int_{c.\forall} d\forall = V_B A_B - 2V_B A_A$$

$$= V_B A_B - 2V_B \frac{A_B}{4}$$

$$= V_B A_B - \frac{V_B A_B}{2}$$

$4A_A = A_B$

$\frac{A_A}{A_B} = \frac{1}{4}$

$$\frac{A_A}{A_B} = \frac{\frac{\pi}{4}\left(\frac{3}{12}\right)^2}{\frac{\pi}{4}\left(\frac{6}{12}\right)^2}$$

rising

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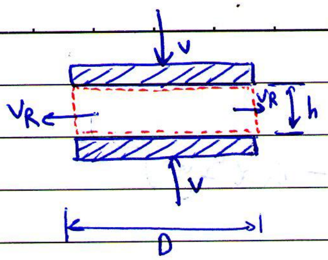
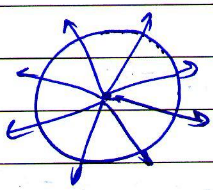



radial convective acceleration?



$$a_R = V_R \frac{dv}{dr} + \frac{\partial V_R}{\partial t}$$

convective local

$$\sum_{c.s} \rho V A = \frac{-d}{dt} \int \rho \, d\forall = 0$$

$$\sum (\cancel{\rho} VA)_i = \sum (\cancel{\rho} VA)_e$$

$$2V \frac{\pi}{4} D^{\cancel{2}} = V_R (\cancel{\pi} \cancel{D} h)$$

$$V_R = \frac{VD}{2h} = \frac{VR}{h} \qquad R = \frac{D}{2}$$

$$a = V_R \frac{\partial v}{\partial r} = \frac{VR}{h} * \frac{V}{h} = \left(\frac{V}{h}\right)^2 R = \frac{D}{2}\left(\frac{V}{h}\right)^2$$

$$V_R = \frac{VR}{h}$$

$$a_{local} = \frac{\partial V_R}{\partial t} \neq 0$$ لأنه الـ h بتتغير مع t

$h = H$ at $t = 0$

at $t = t \Rightarrow h = H - \underline{2Vt}$ تقل بمقدار المسافة التي تقطعها القرصين

$$a = \frac{\partial}{\partial t} \left( \frac{VR}{H - 2Vt} \right)$$

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**5.105** The velocity components of a flow field are given by

$$u = \frac{y}{(x^2+y^2)^{3/2}} \qquad v = \frac{-x}{(x^2+y^2)^{3/2}}$$

Is continuity satisfied? Is the flow irrotational?

$$u = y(x^2+y^2)^{-3/2} \quad , \quad v = -x(x^2+y^2)^{-3/2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \cancel{\frac{\partial w}{\partial z}} = 0$$

$$\frac{\partial u}{\partial x} = \frac{-3}{2}\, 2xy\, (x^2+y^2)^{-5/2} = -3xy\,(x^2+y^2)^{-5/2}$$

$$\frac{\partial v}{\partial y} = (-x) * -\frac{3}{2}\,(2y)(x^2+y^2)^{-5/2} = 3xy\,(x^2+y^2)^{-5/2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

$$0 = 0$$ satisfied continuity equation

$$\boxed{\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}} \neq 0 \qquad \frac{\partial u}{\partial z} = \frac{\partial w}{\partial x} \qquad \frac{\partial v}{\partial z} = \frac{\partial w}{\partial y}$$

0 0

Rotational Flow

**EXAMPLE 5.1 VOLUME FLOW RATE AND MEAN VELOCITY**

Air that has a mass density of 1.24 kg/m$^3$ (0.00241 slugs/ft$^3$) flows in a pipe with a diameter of 30 cm (0.984 ft) at a mass rate of flow of 3 kg/s (0.206 slugs/s). What are the mean velocity and discharge in this pipe for both systems of units?

*Problem Definition*

**Situation:** Airflow in pipe with 30 cm diameter at 3 kg/s.

**Find:**

1. Discharge (m$^3$/s and ft$^3$/s).
2. Mean velocity (m/s and ft/s).

**Assumptions:** Properties are uniformly distributed across section.

**Properties:** $\rho = 1.24$ kg/m$^3$ (0.00241slugs/ft$^3$).

*Plan*

1. Find the volume flow rate using the volume flow rate equation, Eq. (5.5).
2. Calculate the mean velocity using Eq. (5.3).

*Solution*

1. Discharge:

$$Q = \frac{\dot{m}}{\rho} = \frac{3\ \text{kg/s}}{1.24\ \text{kg/m}^3} = \boxed{2.42\ \text{m}^3/\text{s}}$$

$$Q = 2.42\ \text{m}^3/\text{s} \times \left(\frac{35.31\ \text{ft}^3}{1\ \text{m}^3}\right) = \boxed{85.5\ \text{cfs}}$$

2. Mean velocity

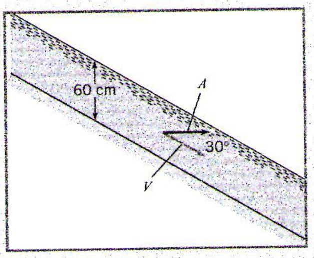
$$V = \frac{Q}{A} = \frac{2.42\ \text{m}^3/\text{s}}{(\frac{1}{4}\pi) \times (0.30\ \text{m})^2} = \boxed{34.2\ \text{m/s}}$$

$$V = 34.2\ \text{m/s} \times \left(\frac{1\ \text{ft}}{0.3048\ \text{m}}\right) = \boxed{112\ \text{ft/s}}$$

**EXAMPLE 5.2 FLOW IN SLOPING CHANNEL**

Water flows in a channel that has a slope of 30°. If the velocity is assumed to be constant, 12 m/s, and if a depth of 60 cm is measured along a vertical line, what is the discharge per meter of width of the channel?

**Sketch:**




*Problem Definition*

**Situation:** Channel slope of 30°. Velocity is 12 m/s and vertical depth is 60 cm.

**Find:** Discharge per meter width (m$^2$/s).

**Assumptions:** Velocity is uniformly distributed across channel.

*Plan*

Use Eq. (5.7) with area based on 1 meter width.

*Solution*

$$\begin{aligned} Q &= \mathbf{V} \cdot \mathbf{A} = V \cos 30 \times A \\ &= 12\ \text{m/s} \times \cos 30° \times 0.6\ \text{m} \\ &= \boxed{6.24\ \text{m}^3/\text{s per meter}} \end{aligned}$$

*Review*

The discharge per unit width is usually designated as $q$.

**EXAMPLE 5.3 DISCHARGE IN CHANNEL WITH NON-UNIFORM VELOCITY DISTRIBUTION**

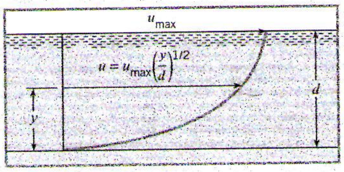
The water velocity in the channel shown in the accompanying figure has a distribution across the vertical section equal to $u/u_{max} = (y/d)^{1/2}$. What is the discharge in the channel if the water is 2 m deep ($d = 2$ m), the channel is 5 m wide, and the maximum velocity is 3 m/s?

*Problem Definition*

**Situation:** Water flows in a 2 m by 5 m channel with a given velocity distribution.

**Find:** Discharge (in m$^3$/s).

**Sketch:**




*Plan*

Find the discharge by using Eq. (5.2).

*Solution*

Discharge equation

$$Q = \int_0^d u\, dA$$

Channel is 5 m wide, so differential area is $dA = 5dy$. Using given velocity distribution,

$$\begin{aligned} Q &= \int_0^2 u_{max}(y/d)^{1/2} 5\, dy \\ &= \frac{5u_{max}}{d^{1/2}} \int_0^2 y^{1/2}\, dy \\ &= \frac{5u_{max}}{d^{1/2}} \frac{2}{3} y^{3/2} \Big|_0^2 \\ &= \frac{5 \times 3}{2^{1/2}} \times \frac{2}{3} \times 2^{3/2} = \boxed{20\ \text{m}^3/\text{s}} \end{aligned}$$

## EXAMPLE 5.6 WATER LEVEL DROP RATE IN DRAINING TANK

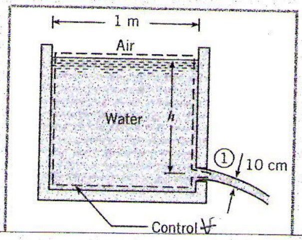
A 10 cm jet of water issues from a 1 m diameter tank. Assume that the velocity in the jet is $\sqrt{2gh}$ m/s where h is the elevation of the water surface above the outlet jet. How long will it take for the water surface in the tank to drop from $h_0 = 2$ m to $h_f = 0.50$ m?

*Problem Definition*

**Situation:** Water draining by a 10 cm jet from 1 m diameter tank.

**Find:** Time (in seconds) to drain from depth of 2 m to 0.5 m.

**Sketch:**




*Plan*

The control selected is shown in the sketch. The control surface is located at and moves with the water surface. Water crosses control surface at location 1.

1. Apply the continuity equation, Eq. (5.25).
2. Analyze term by term.
3. Solve the equation for elapsed time.
4. Calculate time to change levels.

*Solution*

1. Continuity equation

$$\frac{d}{dt}m_{cv} + \sum_{cs}\dot{m}_o - \sum_{cs}\dot{m}_i = 0$$

2. Term-by-term analysis
   - Accumulation rate term

$$dm_{cv} = \rho A_T dh$$

$$\frac{dm_{cv}}{dt} = \rho A_T \frac{dh}{dt}$$

where $A_T$ is cross-sectional area of tank.

- Inlet mass flow rate with no inflow is

$$\sum_{cs}\dot{m}_i = 0$$

- Outlet mass flow rate

$$\sum_{cs}\dot{m}_o = \rho A_1 V_1 = \rho\sqrt{2gh}A_1$$

Substitution of terms in continuity equation:

$$-\rho V_1 A_1 = \frac{d(\rho A_T h)}{dt}$$

$$-\sqrt{2gh}A_1 = A_T\frac{dh}{dt}$$

3. Equation for elapsed time:
   - Separating variables

$$dt = \frac{-A_T}{\sqrt{2g}A_1}\frac{dh}{\sqrt{h}} \quad \text{or} \quad dt = \frac{-A_T}{\sqrt{2g}A_1}h^{-1/2}dh$$

- Integrating

$$t = \frac{-2A_T}{\sqrt{2g}A_1}h^{1/2} + C$$

- Substituting in initial condition, $h(0) = h_0$, and final condition, $h(t) = h_f$, and solving for time

$$t = \frac{2A_T}{\sqrt{2g}A_1}(h_0^{1/2} - h_f^{1/2})$$

4. Time calculation:
   - Evaluating tank and outlet areas

$$A_1 = \frac{\pi}{4}(0.10\text{ m})^2 = 0.01\left(\frac{\pi}{4}\right)\text{m}^2$$

$$A_T = \frac{\pi}{4}D^2 = \frac{\pi}{4}\times(1\text{ m})^2 = \frac{\pi}{4}\text{ m}^2$$

- Elapsed time

$$t = \frac{2(\pi/4)\text{ m}^2}{\sqrt{2\times 9.81\text{ m/s}^2}(\pi/4\times 0.01\text{ m}^2)}(\sqrt{2\text{ m}} - \sqrt{0.5\text{ m}})$$

$$= \boxed{31.9\text{ s}}$$

Subject Date No.

EXAMPLE 5.8 VELOCITY IN A VARIABLE-AREA PIPE

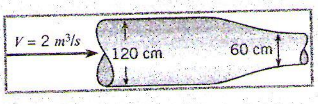
A 120 cm pipe is in series with a 60 cm pipe. The speed of the water in the 120 cm pipe is 2 m/s. What is the water speed in the 60 cm pipe?

*Problem Definition*

**Situation:** Two pipes connected in series.

**Find:** Velocity in 60 cm pipe.

**Sketch:**




*Plan*

Flow rate is the same for each section, $Q_{120} = Q_{60}$. Use Eq. (5.27) to calculate velocity in the 60 cm pipe.

*Solution*

Equation (5.27) for $V_{60}$

$$V_{60} = V_{120}\frac{A_{120}}{A_{60}}$$

Calculation for $V_{60}$:

$$V_{60} = 2\text{m/s} \times \frac{(120\text{ cm})^2}{(60\text{ cm})^2} = \boxed{8\text{ m/s}}$$

EXAMPLE 5.9 WATER FLOW THROUGH A VENTURIMETER

Water with a density of 1000 kg/m$^3$ flows through a vertical venturimeter as shown. A pressure gage is connected across two taps in the pipe (station 1) and the throat (station 2). The area ratio $A_{\text{throat}}/A_{\text{pipe}}$ is 0.5. The velocity in the pipe is 10 m/s. Find the pressure difference recorded by the pressure gage. Assume the flow has a uniform velocity distribution and that viscous effects are not important.

*Problem Definition*

**Situation:** Water flow in venturimeter with gage connected between upstream and throat. Area ratio is 0.5 and pipe velocity is 10 m/s.

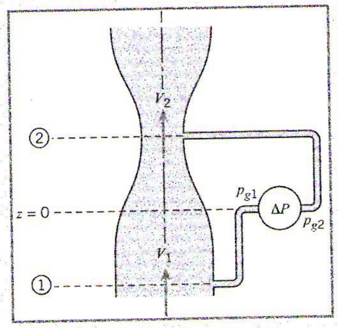
**Find:** Pressure difference measured by gage.

**Assumptions:**

1. Velocity distribution is uniform.
2. Viscous effects are unimportant.

**Properties:** $\rho = 1000$ kg/m$^3$.

**Sketch:**




*Plan*

1. Since viscous effects unimportant, apply the Bernoulli equation between stations 1 and 2.
2. Find mean velocity at station 2 by applying Eq. (5.27), and develop the equation for piezometric pressure.
3. Find the pressure on the gage by applying the hydrostatic equation, Eq. (3.7*a*).

*Solution*

1. The Bernoulli equation

$$p_1 + \gamma z_1 + \rho\frac{V_1^2}{2} = p_2 + \gamma z_2 + \rho\frac{V_2^2}{2}$$

Rewrite the equation in terms of piezometric pressure.

$$p_{z_1} - p_{z_2} = \frac{\rho}{2}(V_2^2 - V_1^2)$$

$$= \frac{\rho V_1^2}{2}\left(\frac{V_2^2}{V_1^2} - 1\right)$$

2. Continuity equation $V_2/V_1 = A_1/A_2$

$$p_{z_1} - p_{z_2} = \frac{\rho V_1^2}{2}\left(\frac{A_1^2}{A_2^2} - 1\right)$$

$$= \frac{1000\text{ kg/m}^3}{2} \times (10\text{ m/s})^2 \times (2^2 - 1)$$

$$= 150\text{ kPa}$$

3. Gage is located at zero elevation. Apply hydrostatic equation through static fluid in gage line between gage attachment point where the pressure is $p_{g_1}$ and station 1 where the gage line is tapped into the pipe,

$$p_{z_1} = p_{g_1}$$

Also $p_{z_2} = p_{g_2}$ so

$$\Delta p_{\text{gage}} = p_{g_1} - p_{g_2} = p_{z_1} - p_{z_2} = \boxed{150\text{ kPa}}$$

Subject Date No.

**EXAMPLE 5.10 APPLICATION OF DIFFERENTIAL FORM OF CONTINUITY EQUATION**

The expression $\boldsymbol{V} = 10x\boldsymbol{i} - 10y\boldsymbol{j}$ is said to represent the velocity for a two-dimensional (planar) incompressible flow. Check to see if the continuity equation is satisfied.

*Problem Definition*

**Situation:** Velocity field is given.

**Find:** Determine if continuity equation is satisfied.

*Plan*

Reduce Eq. (5.33) to two-dimensional flow ($w = 0$ and substitute velocity components into equation).

*Solution*

Continuity equation for two-dimensional flow

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u = 10x; \qquad \frac{\partial u}{\partial x} = 10$$

$$v = -10y; \qquad \frac{\partial v}{\partial y} = -10$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 10 - 10 = 0$$

Continuity is satisfied.

Subject ch6:- momentum equation Date No.

$$\left(\frac{dB}{dt}\right)_{sys} = \frac{d}{dt}\int_{c.v} b\rho\, d\forall + \sum_{c.s} b\rho VA$$

I II III

$$\sum F = ma = m\frac{dv}{dt} = \frac{d\, mv}{dt}$$

$mV = B$

$V = b$

I $\left(\frac{dmV}{dt}\right)_{sys} = \sum F$

II $\frac{d}{dt}\int_{c.\forall} V\rho\, d\forall$

III $\sum V\rho VA = \sum_{c.s} V\dot{m}$

$\int V\rho VA$

$$\sum F = \frac{d}{dt}\int V\rho\, d\forall + \sum V\dot{m}$$

$$\sum F_x = \frac{d}{dt}\int V_x\, \rho\, d\forall + \sum V_x\,\dot{m}$$

$$\sum F_y = \frac{d}{dt}\int V_y\rho\, d\forall + \sum V_y\,\dot{m}$$

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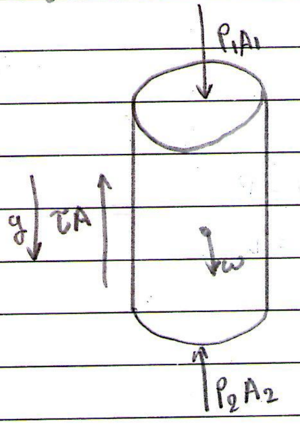
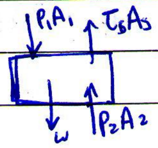
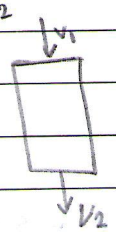
ex:-




① Free body diagram

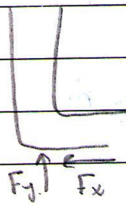


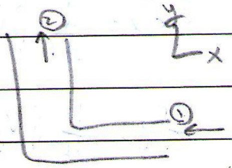

$\Sigma F_z = 0 \Rightarrow \tau(\pi D L) + P_2A_2 - P_1A_1 - W$




$\Sigma V\dot{m} = (-V_2)\dot{m} - (-V_1)\dot{m}$

vector. vector.




$V_{x\,out} = 0$

$V_{x\,in} = -V_1$

$V_{y\,out} = V_2$

$V_{y\,in} = 0$

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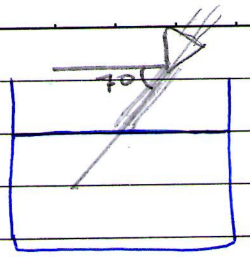
ex)



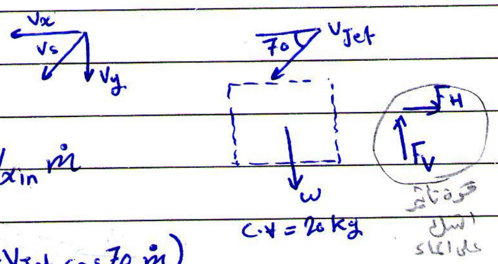

water volume = 20 kg

$V_{Jet} = 20\ m/s$

$d_{Jet} = 30\ mm$

what the supported Force at this time?

Sol :-




$\Sigma F_x = \Sigma V_x \dot{m}$

$F_H = V_{x_{out}} \dot{m} - V_{x_{in}} \dot{m}$

$= 0 - (-V_{Jet} \cos 70\ \dot{m})$

$\boxed{F_H = 96.7\ N}$

$\dot{m} = \rho V A$

$= 1000 * 20 * \frac{\pi}{4} (0.03)^2$

$= 0.1413$

$\Sigma F_z = ??$

$V_{z_o} \dot{m} - V_{z_i} \dot{m}$

$F_V - w = 0 - (-V_{Jet} \sin 70) \rho V_J A_J$

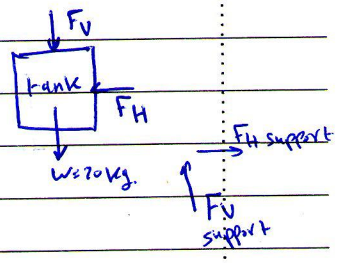
$F_V = w + \rho V_J^2 A_J \sin 70$

$= 9.81 * 20 + 1000 * 20^2 * \frac{\pi}{4} * 0.03^2 \sin 70$

$= 462\ N$

$F_s V = 462 + 20 * 9.81 = 658$

$F_s H = 96.7\ N$



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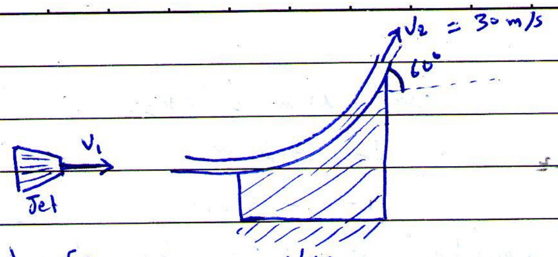
ex)

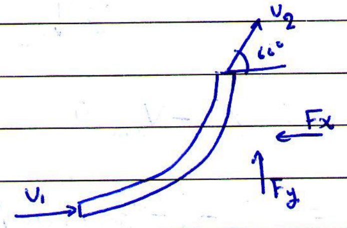



الضغط in و out متساوي

$d_J = 50mm$

$V_J = 36\ m s^{-1}$

Sol :-




$$-F_x = V_2 \cos 60\ (\rho Q) - V_1\ (\rho Q)$$

$$= 30 \cos 60\ \left(1000 * \left(30 * \frac{\pi}{4}(0.05)^2\right)\right) - 36 * \left(1000 * \left(36 * \frac{\pi}{4} * (0.05)^2\right)\right)$$

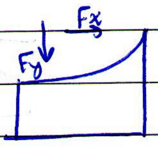
$$\boxed{F_x = 1484\ N}$$

$$F_y = V_2 \sin 60\ \rho \underbrace{V_J A_J}_{\dot{m}} - 0$$

$$= 30 \sin 60 * 1000 * 36 * \frac{\pi}{4} * (0.05)^2$$

$$\boxed{F_y = 1836\ N}$$

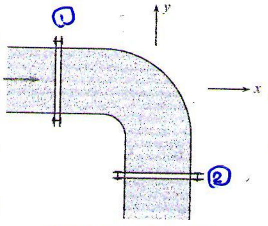
hydrostatic Force On vane




$$\vec{F} = 1484 i - 1836 j$$

Subject Date No.

**6.43** The gage pressure throughout the horizontal 90° pipe bend is 300 kPa. If the pipe diameter is 1 m and the water (at 10°C) flow rate is 10 $m^3/s$, what $x$-component of force must be applied to the bend to hold it in place against the water action?

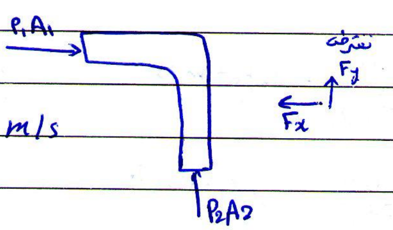


PROBLEMS 6.42, 6.43

$P_1 = 300\ kPa$

$Q = 10\ m^3/s$

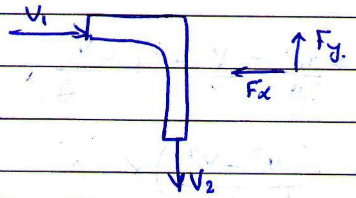
$D = 1\ m$




$Q_1 = Q_2$

$V_1 = V_2 = \frac{Q}{A} = \frac{10}{\frac{\pi}{4}(1)^2} = 12.73\ m/s$

$P_1A_1 - F_x = 0 - V_{x_1}\dot{m}$

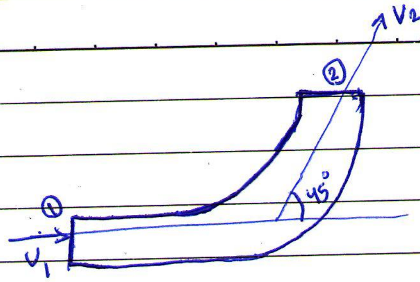



$F_x = P_1A_1 + V_{x_1}\dot{m}$

$= 300\times10^3 \times \frac{\pi}{4}\times1^2 + 12.73 \times 1000 \times 10$

$\boxed{F_x = 363\times10^3\ N}$

ex)



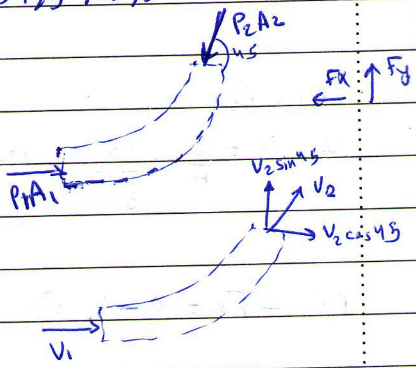

$P_2 = ??$

$D_1 = 600\ mm$

$D_2 = 300\ mm$

$P_1 = 140\ kPa$ gauge

$Q = 0,425\ m^3/s$




$V_1 = \frac{Q}{A_1} = \frac{0,425}{\frac{\pi}{4}(0,6)^2} = 1,503\ m/s$

$V_2 = \frac{Q}{A_2} = \frac{0,425}{\frac{\pi}{4}(0,3)^2} = 6,01\ m/s$

$\frac{P_2}{\gamma} + \cancel{Z_2} + \frac{V_2^2}{2g} = \frac{P_1}{\gamma} + \cancel{Z_1} + \frac{V_1^2}{2g}$

$P_2 = P_1 + \left(\frac{V_1^2 - V_2^2}{2g}\right) \cancel{\gamma}\ \rho$

$P_2 = 140 \times 10^3 + \left(\frac{1,503^2 - 6,01^2}{2}\right) \times 1000 = 123,1\ kPa$

$P_1 A_1 - F_x - P_2 A_2 \cos 45 = V_2 \cos 45 (\rho Q) - V_1 (\rho Q)$

$F_x = P_1 A_1 - P_2 A_2 \cos 45 - V_2 \cos 45\ \dot{m} + V_1\ \dot{m}$

$= 140 \times 10^3 \times \frac{\pi}{4}(0,6)^2 - 123,1 \times 10^3 \times \frac{\pi}{4} \times (0,3)^2 (\cos 45)$

$- 6,01 \cos 45 (425) + 1,503 \times 4,25$

$\boxed{F_x = 32260\ N}$ water على bend [illegible]

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$\Sigma F_y = \Sigma V_y \dot{m}$

$F_y - P_2 A_2 \sin 45 = V_{y_o} \dot{m} - \cancel{V_{y_{in}} \dot{m}}$

$F_y = P_2 A_2 \sin 45 + V_{y_o} \dot{m}$ | $V_{y_o} = V_2 \sin 45$

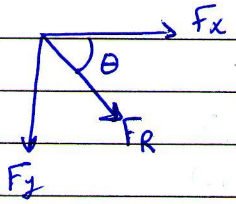
$= 123{,}1 \times 10^3 \times \frac{\pi}{4} \times (0{,}3)^2 \sin 45$

$+ 6{,}01 \times \sin 45 \times 425$

$\boxed{F_y = 7960\, N}$

* Force of the bend.

$\vec{F} = 32260\, i - 7960\, j \; N$




$F_R = \sqrt{F_x^2 + F_y^2} = 33230\, N$

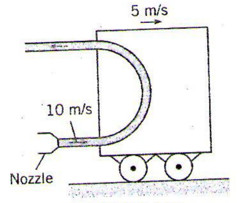
$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{7960}{32260}\right)$

$\theta = 13{,}8°$

N O T E B O O K

Subject | Date | No.

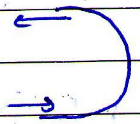
**6.86** A cart is moving along a track at a constant velocity of 5 m/s as shown. Water ($\rho = 1000\ kg/m^3$) issues from a nozzle at 10 m/s and is deflected through 180° by a vane on the cart. The cross-sectional area of the nozzle is $0.0012\ m^2$. Calculate the resistive force on the cart.




PROBLEM 6.86

$V_J = 10\ m/s$

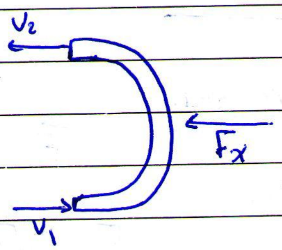
$A_J = 0{,}001\ m^2$



$V_J = 5\ m/s$

نسبة الى العربة

$V_1 = V_2 = 10 - 5 = \underline{\underline{5\ m/s}}$




$-F_x = -V_2 \dot{m} - V_1 \dot{m}$

$-F_x = -\dot{m}(V_1 + V_2)$

$F_x = \dot{m}(V_1 + V_2)$

$= 1000 * 0{,}001 * 5\ (5+5)$

$= 50\ N$

Subject Date No.

× water hammer

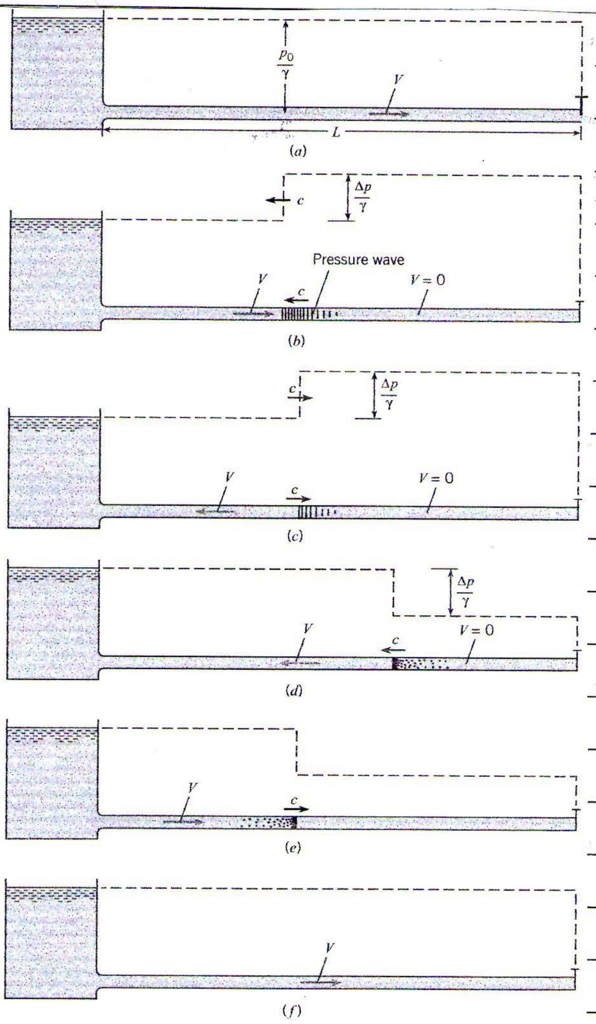



**Figure 6.7**
*Water hammer process.* (a) Initial condition. (b) Condition during time $0 < t < L/c$. (c) Condition during time $L/c < t < 2L/c$. (d) Condition during time $2L/c < t < 3L/c$. (e) Condition during time $3L/c < t < 4L/c$. (f) Condition at time $t = 4L/c$.

زمن الضغط الذي يستغرقه الزمن cyclie time $= \frac{4L}{c}$

الزمن الحرج الخطر critical time $= \frac{2L}{c}$

$E_v = \frac{\Delta P}{\Delta \rho / \rho}$

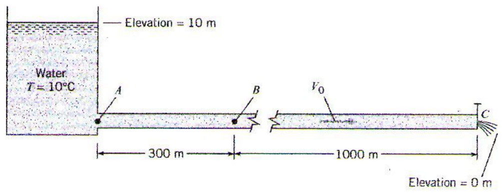
بسبب انغلاق الصمام $\Delta P_{max} = \rho V C$

سرعة الصوت $C = \sqrt{E_v/\rho}$

N O T E B O O K

Subject Date No.

**6.97** The 60-cm pipe carries water (at 10°C) with an initial velocity, $V_0$, of 0.10 m/s. If the valve at $C$ is instantaneously closed at time $t = 0$, what will the pressure-versus-time trace look like at point $B$ for the next 5 s? Graph your results and indicate significant quantitative relations or values from $t = 0$ to $t = 5$ s. What does the pressure versus the position along the pipe look like at $t = 1.5$ s? Plot your results and indicate the velocity or velocities in the pipe.




PROBLEM 6.97

$V = 0,1$ m/s

$C = 1483$ m/s

$$P_{pipe} = 10 \delta - \frac{\rho V^2_{pipe}}{2}$$

$\simeq 98000$ Pa

$\Delta P = \rho V C$

$= 1000 * 0,1 * 1483$

$= 148000$ Pa

$P_{max} = P + \Delta P = \cancel{\ldots}$

$98000 + 148000$

$= 246$ kPa

$P_{min} = P - \Delta P = -50$ kPa

---

ex) $V = 3$ m/s

$L = 10$ km

$t = 10$ s

water hammer التي يولدها تغيير $\Delta P = ??$

$E_V = 2,2 * 10^9$ Pa

Sol:-

$$C = \sqrt{\frac{E_V}{\rho}} = \sqrt{\frac{2,2 * 10^9}{1000}}$$

$= 1483$ m/s

$\Delta P = \rho V C = 1000 * 3 * 1483$

$= 4449000$ Pa

$= 4,449$ MPa

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ex:-) $D = 20\ cm$

$Q = 0.15\ m^3/s$

$\boxed{\frac{4L}{c} = 3\,sec}$

$L = ??$

$C = ??$

sol:-

$C = \sqrt{\frac{E_v}{\rho}} = 1483\ m/s$

$L = \frac{3}{4} C = 1112\ m$ مكان وضع الصمام بعد ٣ ثواني

**EXAMPLE 6.12 PRESSURE RISE DUE TO WATER HAMMER EFFECT**

A rigid pipe leading from a reservoir is 3000 ft long, and water is flowing through it with a velocity of 4 ft/s. If the initial pressure at the downstream end is 40 psig, what maximum pressure will develop at the downstream end when a rapid-acting valve at that end is closed in 1 s?

*Problem Definition*

**Situation:** Water flowing in pipe and valve closed quickly.

**Find:** Maximum pressure (psig) at downstream end.

**Assumptions:** Water temperature is 60°F.

**Properties:** From Table A.5, $E_v = 3.2 \times 10^5$ lbf/in², and $\rho = 1.94$ slugs/ft³.

*Plan*

1. Calculate the speed of sound in the water from Eq. (6.23).
2. Calculate the critical closure time, $t_c$.
3. Check to ensure that valve closure time is less than $t_c$.
4. Calculate pressure rise using Eq. (6.19) and add initial pipe pressure.

*Solution*

1. Calculation for sound speed:

$$c = \sqrt{\frac{E_v}{\rho}} = \sqrt{\frac{320{,}000\ \text{lbf/in}^2 \times 144\ \text{in}^2/\text{ft}^2}{1.94\ \text{slugs/ft}^3}} = 4874\ \text{ft/s}$$

2. Calculation for critical closure time:

$$t_c = 2L/c$$
$$= 2(3000\ \text{ft}/4874\ \text{ft/s}) = 1.23\ \text{s}$$

3. Closure time of 1 s is less than 1.23 s.
4. Pressure rise calculation:

$$\Delta p = \rho V c$$
$$= 1.94\ \text{slugs/ft}^3 \times 4\ \text{ft/s} \times 4874\ \text{ft/s}$$
$$= 37{,}820\ \text{lbf/ft}^2 \times \frac{1\ \text{ft}^2}{144\ \text{in}^2} = 263\ \text{psi}$$

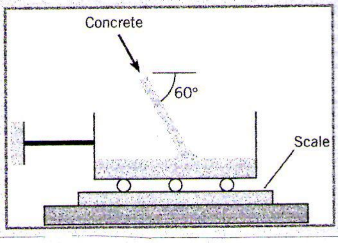
Maximum pressure is

$$p_{max} = 40 + 263 = \boxed{303\ \text{psig}}$$

## EXAMPLE 6.2 CONCRETE FLOWING INTO CART

As shown in the sketch, concrete flows into a cart sitting on a scale. The stream of concrete has a density of $\rho = 150$ lbm/ft$^3$, an area of $A = 1$ ft$^2$, and a speed of $v = 10$ ft/s. At the instant shown, the weight of the cart plus the concrete is 800 lbf. Determine the tension in the cable and the weight recorded by the scale. Assume steady flow.

**Sketch:** Density of concrete, $\rho = 150$ lbm/ft$^3$.




### *Problem Definition*

**Situation:** Concrete flowing into cart held by cable and mounted on a scale.

**Find:**

1. Force (in lbf) on cable.
2. Weight (in lbf) recorded on scale.

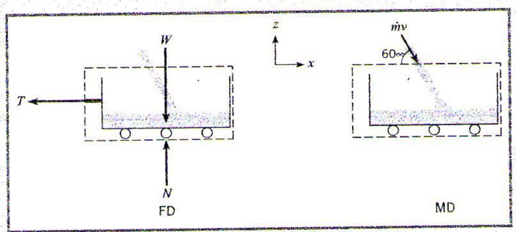
**Assumptions:** The velocity of the concrete in the cart is zero.

### *Plan*

1. Select a control volume that provides force on cable and weight on scale.
2. Sketch the force diagram.
3. Sketch the momentum diagram.
4. Since this problem involves two directions, the component form of the momentum equations in the $x$- and $z$-directions, Eqs. (6.7a) and (6.7c), will be used.
5. Evaluate forces from force diagram.
6. Evaluate momentum terms.
7. Calculate tension in cable and weight on scale.

### *Solution*

1. Control volume selected is shown on diagram. Control volume is stationary.




2. Force diagram shows the tension in the cable and the weight on the scale.
3. Momentum diagram shows only an inflow of momentum. Velocity of the concrete in the tank is neglected.
4. Component momentum equations
   - Momentum equation in $x$-direction

$$\sum F_x = \frac{d}{dt}\int_{cv} v_x \rho \, d\forall + \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix} \qquad \text{(a)}$$

   - Momentum equation in $z$-direction

$$\sum F_z = \frac{d}{dt}\int_{cv} v_z \rho \, d\forall + \sum_{cs} \dot{m}_o v_{oz} - \sum_{cs} \dot{m}_i v_{iz} \qquad \text{(b)}$$

5. Forces from the force diagram

$$\sum F_x = -T$$

$$\sum F_z = N - W$$

6. Evaluation of momentum terms
   - Momentum accumulation: $v_x = 0$, $v_z = 0$, so

$$\frac{d}{dt}\int_{cv} v_x \rho \, d\forall = 0, \ \frac{d}{dt}\int_{cv} v_z \rho \, d\forall = 0.$$

   - Momentum inflow

$$\sum_{cs} \dot{m}_i v_{ix} = \dot{m} v \cos 60° = \rho A v^2 \cos 60°$$

$$\sum_{cs} \dot{m}_i v_{iz} = m(-v \sin 60°) = -\rho A v^2 \sin 60°$$

   - Momentum outflow: No outflow, so,

$$\sum_{cs} \dot{m}_o v_{ox} = 0, \text{ and } \sum_{cs} \dot{m}_o v_{oz} = 0.$$

7. Evaluate tension in cable using (a).

$$-T = -\rho A v^2 \cos 60°$$

$$T = (150 \text{ lbm/ft}^3)\left(\frac{\text{slugs}}{32.2 \text{ lbm}}\right)(1 \text{ ft}^2)(10 \text{ ft/s})^2 \cos 60°$$

$$= \boxed{233 \text{ lbf}}$$

Evaluate force on scale using (b).

$$N - W = -(-\rho A v^2 \sin 60°)$$

$$N = W + \rho A v^2 \sin 60°$$

$$= 800 \text{ lbf} + 403 \text{ lbf} = \boxed{1200 \text{ lbf}}$$

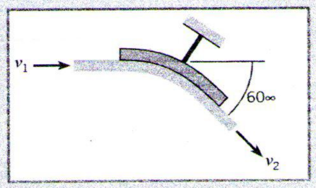
### *Review*

1. The weight recorded by the scale is larger than the weight of the cart because of the momentum carried by the fluid jet.
2. Notice that unit conversions are usually needed when using English units.
3. There will be some velocity in the cart due to mixing, but the momentum associated with those velocities would be insignificant, so the momentum accumulation term can be neglected.
4. Answers are expressed with three significant figures.

**EXAMPLE 6.4 WATER DEFLECTED BY A VANE**

A water jet is deflected 60° by a stationary vane as shown in the figure. The incoming jet has a speed of 100 ft/s and a diameter of 1 in. Find the force exerted by the jet on the vane. Neglect the influence of gravity.

**Sketch:**




*Problem Definition*

**Situation:** Water deflected by a vane.

**Find:** Force (lbf) on vane due to jet.

**Assumptions:**

1. Viscous effects are negligible.
2. Neglect gravitational effects.

**Properties:** $\rho = 62.4\ \text{lbm/ft}^3 = 1.94\ \text{slug/ft}^3$.

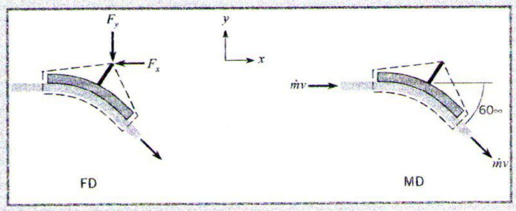
*Plan*

From the Bernoulli equation, since the pressure is constant, the inlet and outlet speeds are the same. Also, from continuity, $\dot{m}_1 = \dot{m}_2 = \dot{m}$.

1. Select a control volume such that the control surface includes the force on the vane and flux of momentum.
2. Sketch the force diagram.
3. Sketch the momentum diagram.
4. Use the vector form of momentum equation, Eq. (6.6).
5. Evaluate force terms.
6. Evaluate momentum terms
7. Evaluate mass flow rate.
8. Calculate force.

*Solution*

1. The control volume selected is shown in the sketch. The control volume is stationary.




2. The force diagram shows only the reaction force.
3. The momentum diagram shows an inflow and outflow.
4. Vector form of momentum equation.

$$\sum \boldsymbol{F} = \frac{d}{dt}\int_{cv} \rho \boldsymbol{v}\, d\forall + \sum_{cs} \dot{m}_o \boldsymbol{v}_o - \sum_{cs} \dot{m}_i \boldsymbol{v}_i$$

5. Force vector is

$$\sum \boldsymbol{F} = -F_x\mathbf{i} - F_y\mathbf{j}$$

6. Evaluation of momentum terms
   - Control volume is stationary, $\frac{d}{dt}\int_{cv} \rho \boldsymbol{v}\, d\forall = 0$
   - Momentum outflow vector,
     $$\sum_{cs} \dot{m}_o \boldsymbol{v}_o = [(\dot{m}v\cos 60°)\mathbf{i} - (\dot{m}v\sin 60°)\mathbf{j}].$$
   - Momentum inflow vector, $\sum_{cs} \dot{m}_i \boldsymbol{v}_i = \dot{m}v\mathbf{i}$.
7. Mass flow rate
   $$\dot{m} = \rho A v$$
   $$= (1.94\ \text{slug/ft}^3)(\pi \times 0.0417^2\ \text{ft}^2)(100\ \text{ft/s})$$
   $$= 1.06\ \text{slug/s}$$
8. Force
   $$-F_x\mathbf{i} - F_y\mathbf{j} = (\dot{m}v\cos 60° - \dot{m}v)\mathbf{i} - (\dot{m}v\sin 60°)\mathbf{j}$$
   For each component,
   $$-F_x = \dot{m}v\cos 60° - \dot{m}v$$
   $$-F_y = -\dot{m}v\sin 60°$$
   Force in $x$-direction
   $$F_x = \dot{m}v(1 - \cos 60°)$$
   $$= (1.06\ \text{slug/s})(100\ \text{ft/s})(1 - \cos 60°)$$
   $$F_x = \boxed{53.0\ \text{lbf}}$$
   Force in $y$-direction
   $$F_y = \dot{m}v\sin 60°$$
   $$= (1.06\ \text{slug/s})(100\ \text{ft/s})\sin 60°$$
   $$F_y = \boxed{91.8\ \text{lbf}}$$
   The force of the jet on the vane ($\mathbf{F}_{jet}$) is opposite in direction to the force required to hold the vane stationary ($\mathbf{F}$). Therefore,
   $$\mathbf{F}_{jet} = (53.0\ \text{lbf})\mathbf{i} + (91.8\ \text{lbf})\mathbf{j}$$

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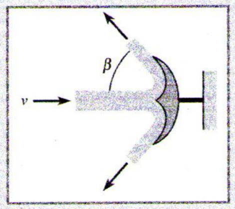
**EXAMPLE 6.5 FORCE ON AN AXISYMMETRIC VANE**

As shown in the figure, an incident jet of fluid with density ρ, speed $v$, and area $A$ is deflected through an angle β by a stationary, axisymmetric vane. Find the force required to hold the vane stationary. Express the answer using ρ, $v$, $A$, and β. Neglect the influence of gravity.

**Sketch:** Gravitational effects are negligible.




*Problem Definition*

**Situation:** Fluid deflected by axisymmetric vane.

**Find:** Force required to hold vane stationary.

**Assumptions:**

1. Flow is steady.
2. Fluid is incompressible.
3. Viscous effects are negligible.

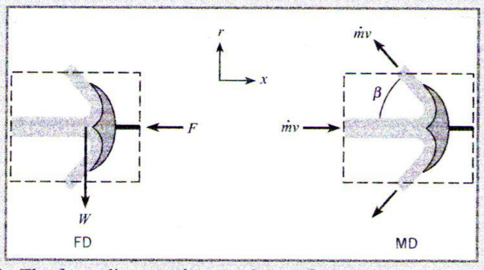
*Plan*

Because the pressure is constant, the Bernoulli equation shows the inlet and outlet speeds are the same. Application of the continuity equation shows the inlet and outlet mass flows are also the same.

1. Select a control volume with the constraining force on control surface.
2. Sketch the force diagram.
3. Sketch the momentum diagram.
4. Apply the component form of the momentum equation in $x$-direction, Eq. (6.7a).
5. Evaluate force terms.
6. Evaluate momentum terms.
7. Calculate force.

*Solution*

1. Control volume selected is shown. Control volume is stationary.




2. The force diagram shows only one force.
3. The momentum diagram shows one momentum flux in and one axisymmetric flux out. The net radial flux of momentum is zero, so only the component in the axial direction contributes to the momentum flux.
4. Momentum equation in $x$-direction.

$$\sum F_x = \frac{d}{dt}\int_{cv} v_x \rho \, d\forall + \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix}$$

5. Sum of forces

$$\sum F_x = -F$$

6. Evaluation of momentum terms
   - Accumulation term for stationary control volume is
   $$\frac{d}{dt}\int_{cv} v_x \rho \, d\forall = 0.$$
   - Momentum outflow is $\sum_{cs} \dot{m}_o v_{ox} = -\dot{m} v \cos\beta$.
   - Momentum inflow is $\sum_{cs} \dot{m}_i v_{ix} = \dot{m} v$.

7. Force on vane

$$-F = -\dot{m}v(1 + \cos\beta)$$
$$F = \dot{m}v(1 + \cos\beta)$$

Apply mass flow rate equation, $\dot{m} = \rho A v$,

$$\boxed{F = \rho A v^2 (1 + \cos\beta)}$$

and the direction of this force is to the left, as shown in the force diagram.

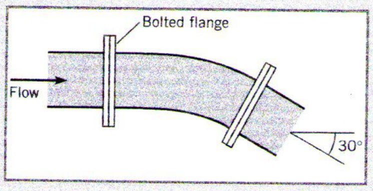
*Review*

This type of reverse flow vane is used to reverse thrust on aircraft engines.

### EXAMPLE 6.6 FORCES ACTING ON A PIPE BEND

A 1 m–diameter pipe bend shown in the diagram is carrying crude oil ($S = 0.94$) with a steady flow rate of $2\ \text{m}^3/\text{s}$. The bend has an angle of 30° and lies in a horizontal plane. The volume of oil in the bend is $1.2\ \text{m}^3$, and the empty weight of the bend is 4 kN. Assume the pressure along the centerline of the bend is constant with a value of 75 kPa gage. Find the net force required to hold the bend in place.

**Sketch:**




*Problem Definition*

**Situation:** Crude oil flows through a 30° pipe bend.

**Find:** Force (in kN) required to hold bend in place.

**Assumptions:** Pressure is constant through bend.

**Properties:** $S_{\text{oil}} = 0.94$.

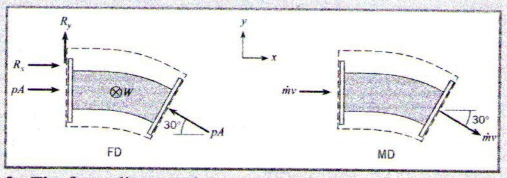
*Plan*

From the continuity equation, the inlet and outlet mass flows are the same.

1. Select a control volume that accommodates the pressure forces and reaction forces at the flanges.
2. Sketch the force diagram.
3. Sketch the momentum diagram.
4. Use the vector form of the momentum equation, Eq. (6.6).
5. Evaluate the sum of the forces.
6. Evaluate the momentum terms.
7. Calculate the reaction force.

*Solution*

1. The control volume selected is shown. The control volume is stationary. The $z$-direction is outward from the page.




2. The force diagram shows pressure forces and the component reaction forces.
3. Vector form of momentum equation

$$\sum \boldsymbol{F} = \frac{d}{dt}\int_{\text{cv}} \rho \mathbf{v}\, d\mathcal{V} + \sum_{\text{cs}} \dot{m}_o \mathbf{v}_o - \sum_{\text{cs}} \dot{m}_i \mathbf{v}_i$$

4. Sum of the forces: The weight of the pipe and fluid therein is $W$ and acts in the negative $z$-direction.

$$\sum \boldsymbol{F} = (R_x + pA - pA\cos 30°)\mathbf{i} + (R_y + pA\sin 30°)\mathbf{j} + (R_z - W)\mathbf{k}$$

5. Momentum terms
   - Accumulation term for stationary control volume is $\frac{d}{dt}\int_{\text{cv}} \rho \mathbf{v}\, d\mathcal{V} = 0$.
   - Momentum outflow is $\sum_{\text{cs}} \dot{m}_o \mathbf{v}_o = (\dot{m}v\cos 30°)\mathbf{i} - (\dot{m}v\sin 30°)\mathbf{j}$.
   - Momentum inflow is $\sum_{\text{cs}} \dot{m}_i \mathbf{v}_i = (\dot{m}v)\mathbf{i}$.

6. Reaction force

$$(R_x + pA - pA\cos 30°)\mathbf{i} + (R_y + pA\sin 30°)\mathbf{j} + (R_z - W)\mathbf{k} = [\dot{m}v(\cos 30 - 1)]\mathbf{i} - (\dot{m}v\sin 30°)\mathbf{j}$$

   - Equating components

$$R_x + pA - pA\cos 30° = \dot{m}v\cos 30° - \dot{m}v$$
$$R_y + pA\sin 30° = -\dot{m}v\sin 30°$$
$$R_z - W = 0$$

   - Pressure force

$$pA = (75\ \text{kN/m}^2)(\pi \times 0.5^2\ \text{m}^2) = 58.9\ \text{kN}$$

   - Fluid speed

$$v = Q/A = \frac{(2\ \text{m}^3/\text{s})}{(\pi \times 0.5^2\ \text{m}^2)} = 2.55\ \text{m/s}$$

   - Momentum flux

$$\dot{m}v = \rho Q v = (0.94 \times 1000\ \text{kg/m}^3)(2\ \text{m}^3/\text{s})(2.55\ \text{m/s}) = 4.79\ \text{kN}$$

Reaction force in $x$-direction

$$R_x = -(pA + \dot{m}v)(1 - \cos 30°) = -(58.9 + 4.79)(\text{kN})(1 - \cos 30°) = \boxed{-8.53\ \text{kN}}$$

Reaction force in $y$-direction

$$R_y = -(pA + \dot{m}v)\sin 30° = -(58.9 + 4.79)(\text{kN})(\sin 30°) = \boxed{-31.8\ \text{kN}}$$

Reaction force in $z$-direction. (The bend weight includes the oil plus the empty pipe).

$$W = \gamma \mathcal{V} + 4\ \text{kN} = (0.94 \times 9.81\ \text{kN/m}^3)(1.2\ \text{m}^3) + 4\ \text{kN} = \boxed{15.1\ \text{kN}}$$

Reaction force vector

$$\mathbf{R} = (-8.53\ \text{kN})\mathbf{i} + (-31.8\ \text{kN})\mathbf{j} + (15.1\ \text{kN})\mathbf{k}$$

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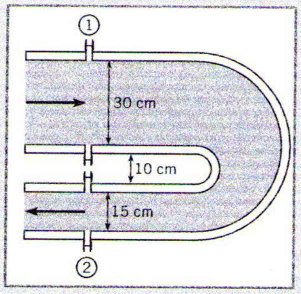
### EXAMPLE 6.7 WATER FLOW THROUGH REDUCING BEND

Water flows through a 180° reducing bend, as shown. The discharge is $0.25\ m^3/s$, and the pressure at the center of the inlet section is 150 kPa gage. If the bend volume is $0.10\ m^3$, and it is assumed that the Bernoulli equation is valid, what force is required to hold the bend in place? The metal in the bend weighs 500 N. The water density is $1000\ kg/m^3$. The bend is in the vertical plane.

**Sketch:**




*Problem Definition*

**Situation:** Water flow through reducing bend.

**Find:** Force (in newtons) required to hold bend in place.

**Assumptions:**

1. The Bernoulli equation is valid.
2. Neglect pipe wall thickness.

**Properties:** $\rho = 1000\ kg/m^3$.

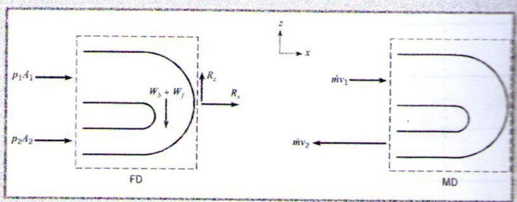
*Plan*

The flow is steady, so $Q_1 = Q_2 = Q$.

1. Select control volume that encloses bend and the reaction force acts on the control surface.
2. Sketch the force diagram.
3. Sketch the momentum diagram.
4. Apply the component form of the momentum equation in the $x$- and $z$-directions, Eqs. (6.7a) and (6.7c).
5. Evaluate the force terms.
6. Evaluate the momentum terms.
7. Solve momentum equations for reaction forces.
8. Calculate the inlet and outlet speed.
9. Apply the Bernoulli equation to find the outlet pressure.
10. Calculate the reaction force.

*Solution*

1. The control volume selected is shown. The control volume is stationary.




2. There are two forces due to pressure and a reaction force component in the $x$-direction, and there are weight and reaction forces component in the $z$-direction.
3. There is inlet and outlet momentum flux in $x$-direction
4. Momentum equations in $x$- and $z$-directions

$$\sum F_x = \frac{d}{dt}\int_{cv} v_x \rho\, d\forall + \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix}$$

$$\sum F_z = \frac{d}{dt}\int_{cv} v_z \rho\, d\forall + \sum_{cs} \dot{m}_o v_{oz} - \sum_{cs} \dot{m}_i v_{iz}$$

5. Summation of forces in $x$- and $z$-directions

$$\sum F_x = p_1 A_1 + p_2 A_2 + R_x$$

$$\sum F_z = R_z - W_b - W_f$$

6. Evaluation of momentum terms
   - Accumulation terms, steady flow

$$\frac{d}{dt}\int_{cv} v_x \rho\, d\forall = 0, \text{ and } \frac{d}{dt}\int_{cv} v_z \rho\, d\forall = 0.$$

   - Momentum outflow

$$\sum_{cs} \dot{m}_o v_{ox} = \dot{m}(-v_2) = -\rho Q v_2, \text{ and } \sum_{cs} \dot{m}_o v_{oz} = 0.$$

   - Momentum inflow

$$\sum_{cs} \dot{m}_i v_{ix} = \dot{m} v_1 = \rho Q v_1, \text{ and } \sum_{cs} \dot{m}_i v_{iz} = 0.$$

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7. Solution for reaction forces
   - $x$-direction

$$p_1A_1 + p_2A_2 + R_x = -\rho Q(v_2 + v_1)$$

$$R_x = -(p_1A_1 + p_2A_2) - \rho Q(v_2 + v_1)$$

   - $z$-direction

$$R_z = W_b + W_f$$

8. Inlet and outlet speeds

$$v_1 = \frac{Q}{A_1} = \frac{0.25\ \text{m}^3/\text{s}}{\pi/4 \times 0.3^2\ \text{m}^2} = 3.54\ \text{m/s}$$

$$v_2 = \frac{Q}{A_2} = \frac{0.25\ \text{m}^3/\text{s}}{\pi/4 \times 0.15^2\ \text{m}^2} = 14.15\ \text{m/s}$$

9. Outlet pressure (the Bernoulli equation between sections 1 and 2)

$$p_1 + \frac{\rho v_1^2}{2} + \gamma z_1 = p_2 + \frac{\rho v_2^2}{2} + \gamma z_2$$

From diagram, neglecting pipe wall thickness, $z_1 - z_2 = 0.325$ m.

$$p_2 = p_1 + \frac{\rho(v_1^2 - v_2^2)}{2} + \gamma(z_1 - z_2)$$

$$= 150\ \text{kPa} + \frac{(1000)(3.54^2 - 14.15^2)\text{Pa}}{2}$$

$$+ (9810)(0.325)\text{Pa}$$

$$= 59.3\ \text{kPa}$$

10. Reaction force
   - Pressure forces

$$p_1A_1 + p_2A_2 = (150\ \text{kPa})(\pi \times 0.3^2/4\ \text{m}^2)$$

$$+ (59.3\ \text{kPa})(\pi \times 0.15^2/4\ \text{m}^2)$$

$$= 11.6\ \text{kN}$$

   - Momentum flux

$$\rho Q(v_2 + v_1) = (1000\ \text{kg/m}^3)(0.25\ \text{m}^3)$$

$$\times (14.15 + 3.54)(\text{m/s})$$

$$= 4420\ \text{N}$$

   - Reaction force components

$$R_x = -(11.6\ \text{kN}) - (4.42\ \text{kN})$$

$$= \boxed{-16.0\ \text{kN}}$$

$$R_z = W_b + W_f$$

$$= 500\ \text{N} + (9810\ \text{N/m}^3)(0.1\ \text{m}^3)$$

$$= \boxed{1.48\ \text{kN}}$$

N O T E B O O K

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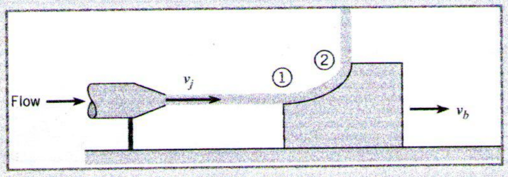
**EXAMPLE 6.10 JET IMPINGING ON MOVING BLOCK**

A stationary nozzle produces a water jet with a speed of 50 m/s and a cross-sectional area of 5 $cm^2$. The jet strikes a moving block and is deflected 90° relative to the block. The block is sliding with a constant speed of 25 m/s on a surface with friction. The density of the water is 1000 $kg/m^3$. Find the frictional force $F$ acting on the block.

Solve the problem using two different inertial reference frames: (a) the moving block and (b) the stationary nozzle.

**Sketch:**




*Problem Definition*

**Situation:** Jet impinges on block moving at constant velocity.

**Find:** The force (in newtons) on the block using

(a) the block as the inertial reference frame.

(b) the nozzle as the inertial reference frame.

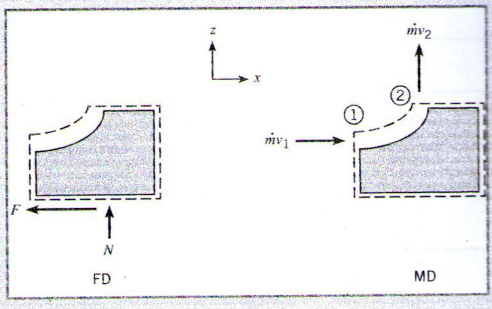
*Plan*

Two different inertial reference frames will be used. Case (a) will use the moving cart, which is a valid inertial frame because it moves at a constant velocity. Case (b) will use the stationary nozzle location.

1. Select a control volume that moves with the block.
2. Sketch the force diagram.
3. Sketch the momentum diagram.
4. Apply the component form of the momentum equation in the $x$-direction, Eq. (6.7a).
5. Evaluate the sum of the forces.
6. Evaluate the momentum terms using (a) the moving block and (b) the stationary nozzle as inertial reference frames.
7. Evaluate mass flow rate.
8. Calculate force on cart.

*Solution*

1. The control volume selected is shown in the sketch. The control volume is not stationary.




2. The force diagram shows one force in the horizontal direction.
3. The momentum diagram shows an influx and outflux of momentum.

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4. Momentum equation

$$\sum F_x = \frac{d}{dt}\int_{cv} v_x \rho\, d\forall + \sum_{cs} \dot{m}_o v_{ox} - \sum_{cs} \dot{m}_i v_{ix}$$

5. The sum of the forces

$$\sum F_x = -F$$

6. Evaluation of terms in momentum equation
   (a) Inertial reference frame on cart

- Accumulation term with $v_x = 0$ is $\frac{d}{dt}\int_{cv} v_x \rho\, d\forall = 0$.
- Momentum inflow for $x$-component of velocity at station 1, $v_{ix} = v_j - v_b$, is

$$\sum_{cs} \dot{m}_i v_{ix} = \dot{m}(v_j - v_b)$$

- Momentum outflow for $x$-component of velocity at station 2, $v_{ox} = 0$, is

$$\sum_{cs} \dot{m}_o v_{ox} = 0$$

   (b) Inertial reference frame at nozzle

- Accumulation term with $v_x = v_b =$ constant, is

$$\frac{d}{dt}\int_{cv} v_x \rho\, d\forall = 0.$$

- Momentum inflow for $x$-component of velocity at station 1, $v_{xi} = v_j$, is

$$\sum_{cs} \dot{m}_i v_{ix} = \dot{m} v_j$$

- Momentum outflow for $x$-component of velocity at station 2, $v_{xo} = v_b$, is

$$\sum_{cs} \dot{m}_o v_{ox} = \dot{m} v_b$$

7. Mass flow rate. Since flow is steady with respect to the block, $\dot{m}_i = \dot{m}_o = \dot{m}$.

$$\dot{m} = \rho A(v_j - v_b)$$

8. Evaluate force.
   (a) Moving block as inertial reference frame

$$-F = -\rho A(v_j - v_b)^2$$
$$F = \rho A(v_j - v_b)^2$$

   (b) Stationary nozzle as inertial reference frame

$$-F = \dot{m} v_b - \dot{m} v_j = -\dot{m}(v_j - v_b)$$
$$F = \rho A(v_j - v_b)^2$$

Force on cart

$$F_x = (1000\ \text{kg/m}^3)(5 \times 10^{-4}\ \text{m}^2)(50 - 25)^2 (\text{m/s})^2$$
$$F_x = \boxed{312\ \text{N}}$$

*Review*

Note that the same answer for force is obtained independent of the inertial reference frame chosen.

**EXAMPLE 6.11 PROPELLANT MASS RATIO FOR ACHIEVING ORBITAL VELOCITY**

A single-stage rocket utilizing a liquid oxygen/kerosene propellant has a specific impulse of 3200 m/s. The orbital velocity for an earth satellite is 7600 m/s. What would be the ratio of propellant mass to total initial mass to achieve orbital velocity?

*Problem Definition*

**Situation:** Rocket launch to achieve orbital velocity.

**Find:** Ratio of propellent mass to initial mass.

*Plan*

1. Use Eq. (6.18) to calculate initial/final mass ratio.
2. Calculate the propellant/initial mass ratio using $m_p = m_i - m_f$.

*Solution*

1. From Eq. (6.18)

$$v_{bo} = I_{sp} \ln \frac{m_i}{m_f}$$

$$\frac{m_i}{m_f} = \exp\left(\frac{v_{bo}}{I_{sp}}\right) = \exp\left(\frac{7600}{3200}\right) = 10.7$$

2. Solve for propellant/initial mass ratio:

$$\frac{m_p}{m_i} = 1 - \frac{m_f}{m_i}$$
$$= 1 - \frac{1}{10.7} = \boxed{0.906}$$

*Review*

For single-stage rockets, a very large fraction of the initial mass must be propellant to achieve orbital speeds. For this reason, multi-stage rockets are used in space applications.

Subject ch.7 Energy principle Date No.

energy → stored energy ⇒ P.E, K.E, I.E

energy → transiation energy ⇒ heat, work

$\Delta E = Q - W$ Q:- heat transfered to the system
W:- work done by the system

$E = K.E + P.E + \underline{u}$ → الطاقة الناتجة عن الذرات

* Q تكون موجبة اذا اضفنا حرارة للنظام والعكس صحيح
* W تكون موجبة اذا عمل النظام شغل وسالبة اذا عُمِل على النظام شغل

** $\frac{dE}{dt} = \dot{Q} - \dot{W}$

$\dot{W}$ → rate of producing work (J/s)

$\dot{Q}$ → rate of heat transfer (J/s)

$\frac{dE}{dt}$ → rate of chang energy of the system

** Energy equation:-

$$\frac{P_1}{\gamma} + \frac{\alpha_1 V_1^2}{2g} + Z_1 + h_p = \frac{P_2}{\gamma} + \frac{\alpha_2 V^2}{2g} + Z_2 + h_L + h_t$$

* معنى $\alpha_1$ ، $\alpha_2$ (kinetic energy factor) ⇒ uniform flow ⇒ $\alpha_1 = \alpha_2 = 1$

nonuniform ⇒ $\alpha_1 > 1$ , $\alpha_2 > 1$

$h_p$ : ارتفاع السائل فوق المضخة (head given by the pump to the fluid)

$h_t$ : " " فوق التوربين ( " " " " turbine " " " " )

$h_L$ : ارتفاع السائل المفقود نتيجة اللزوجة (head loses due to viscous action)

NOTEBOOK

Subject Date No.

* يمكن حساب $\alpha$ عن طريق ← اعطى سرعة $v$ ⇐ $\alpha = \frac{1}{A}\int_A \left(\frac{v}{\bar{V}}\right)^3 dA$

⇐ السرعة المتوسطة $\bar{V}$

* لحساب قدرة المضخة $\underset{\text{power}}{W_p} = \gamma Q h_p$

* لحساب قدرة التوربين $W_t = \gamma Q h_t$

* تعد معادلة برنولي حالة خاصة من Energy equ. حيث أن برنولي :-

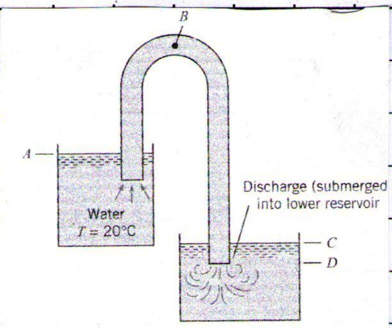
① لا يوجد مضخة ② لا يوجد توربين ③ لا تأثير لقوى اللزوجة ④ steady

⑤ $\alpha_1 = \alpha_2 = 1$

Subject Date No.

**7.33** For this siphon the elevations at $A$, $B$, $C$, and $D$ are 30 m, 32 m, 27 m, and 26 m, respectively. The head loss between the inlet and point $B$ is three-quarters of the velocity head, and the head loss in the pipe itself between point $B$ and the end of the pipe is one-quarter of the velocity head. For these conditions, what is the discharge and what is the pressure at point $B$? The pipe diameter = 25 cm. Assume $\alpha = 1.0$ at all locations.

**7.34** For this system, point $B$ is 10 m above the bottom of the upper reservoir. The head loss from $A$ to $B$ is $1.8V^2/2g$, and the pipe area is $10^{-4}$ m$^2$. Assume a constant discharge of $8 \times 10^{-4}$ m$^3$/s. For these conditions, what will be the depth of water in the upper reservoir for which cavitation will begin at point $B$? Vapor pressure = 1.23 kPa and atmospheric pressure = 100 kPa. Assume $\alpha = 1.0$ at all locations.




PROBLEMS 7.33, 7.34

7.33) ① discharge ② pressure at point B

sol:-

* head loses :-

$$h_{L_{pipe}} = \frac{V_p^2}{2g}$$

$$h_{total} = h_{pipe} + h_{L_{outlet}} = \frac{2V_p^2}{2g}$$

* energy equation (A → C)

$$\cancel{\frac{P_A}{\gamma}}_0 + \cancel{\frac{\alpha_A V_A^2}{2g}}_0 + Z_A = \cancel{\frac{P_C}{\gamma}}_0 + \cancel{\frac{\alpha_C V_C^2}{2g}}_0 + Z_C + h_L$$

$$0 + 0 + 30 = 0 + 0 + 27 + \frac{2V_p^2}{2g}$$

$$\boxed{V_p = 5,42\ m/s}$$

* Flowrate equation :-

$$Q = V_p A_p$$

$$= 5,42 * \left(\frac{\pi}{4} * 0,25^2\right) = 0,226\ m^3/s$$

Subject Date No.

* energy equation $(A \to B)$

$$\cancel{\frac{P_A}{\gamma}}_0 + \cancel{\frac{\alpha_1 V_A^2}{2g}}_0 + Z_A = \frac{P_B}{\gamma} + \cancel{\frac{\alpha_2 V_B^2}{2g}} + Z_B + h_L$$

$$30 = \frac{P_B}{\gamma} + \frac{V_P}{2g} + 32 + 0{,}75\frac{V_P^2}{2g}$$

$$\frac{P_B}{\gamma} = -2 - 1{,}75 \times 1{,}497$$

$$\boxed{P_B = -46{,}3 \text{ kPa. gauge}}$$

7,34)

* Flowrate equ. $\Rightarrow V = Q/A$

$= 8 \text{ m/s}$

& $\frac{V^2}{2g} = \frac{8^2}{2 \times 9{,}81} = 3{,}262 \text{ m}$

$$h_{L(A \to B)} = \frac{1{,}8 \; V^2}{2g} = 5{,}872 \text{ m}$$

& energy equation :- $A \to B$ ( $Z = 0$ at bottom of reservoir

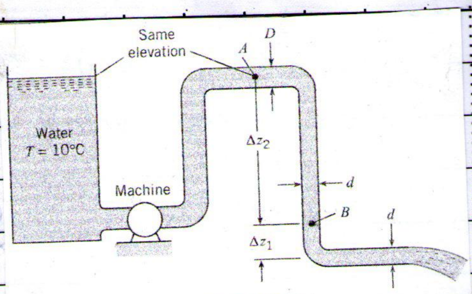
$$\frac{P_A}{\gamma} + \frac{V_A^2}{2g} + Z_A = \frac{P_B}{\gamma} + \frac{V_B^2}{2g} + Z_B + h_L$$

$$\frac{100000}{9810} + 0 + Z_A = \frac{1230}{9810} + 3{,}262 + 10 + 5{,}872$$

$$Z_A = \text{depth} = 9{,}07 \text{ m}$$

Subject Date No.

**7.35** In this system, $d = 6$ in., $D = 12$ in., $\Delta z_1 = 6$ ft, and $\Delta z_2 = 12$ ft. The discharge of water in the system is 10 cfs. Is the machine a pump or a turbine? What are the pressures at points $A$ and $B$? Neglect head losses. Assume $\alpha = 1.0$ at all locations.




PROBLEM 7.35

* pump

$$V_2 = \frac{Q}{A_2} = \frac{10}{\frac{\pi}{4}\times\left(\frac{6}{12}\right)} = 50{,}93 \text{ ft/s}$$

take Pipe exit ⇒ Zero evaluation

$$z_1 + h_p = \frac{V_2^2}{2g} + z_2$$

$$h_p = \frac{V_2^2}{2g} - (z_1 + z_2) = \underline{22{,}3 \text{ ft}}$$

* energy equ. B → exit

$$\frac{P_B}{\gamma} + z_B = z_2$$

$$P_B = \gamma(z_2 - z_B)$$

$$= 62{,}4(-6) = -374 \text{ psfg}$$

$$\boxed{P_B = 2{,}6 \text{ psig}}$$

$$V_A = \left(\frac{6}{12}\right)^2 \times 50{,}93 = 12{,}73 \text{ ft/s}$$

* energy equ A → exit

$$\frac{P_A}{\gamma} + z_A + \frac{V_A^2}{2g} = \frac{V_2^2}{2g}$$

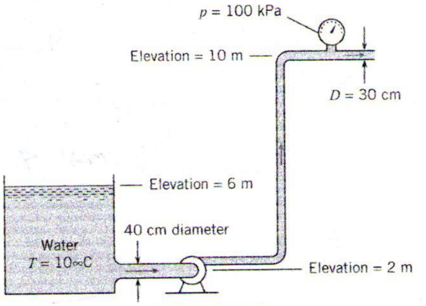
$$P_A = \gamma\left(\frac{V_2^2}{2g} - z_A - \frac{V_A^2}{2g}\right)$$

$$P_A = 1233 \text{ psfg} = 8{,}56 \text{ psig}$$

N O T E B O O K

Subject Date No.

7.50 Water (10°C) is flowing at a rate of 0.25 $m^3/s$, and it is assumed that $h_L = 2V^2/2g$ from the reservoir to the gage, where $V$ is the velocity in the 30-cm pipe. What power must the pump supply? Assume $\alpha = 1.0$ at all locations.




PROBLEM 7.50

$E = Z$

elevation

$V = \frac{Q}{A}$

$$\cancel{\frac{P_A}{\gamma}}_{0} + \cancel{\alpha_1 \frac{V_A^2}{2g}}_{0} + Z_A + h_p = P_B + \alpha_2 \frac{V_B^2}{2g} + Z_B + \cancel{h_t}_{0} + h_L$$

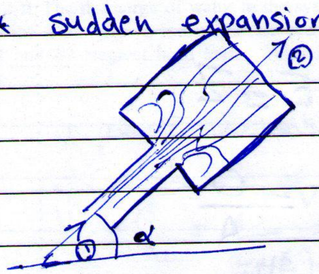
$$\Rightarrow 6 + h_p = \frac{100 \times 10^3}{9810} + \frac{\left(\frac{0.25}{\frac{\pi}{4}(0.3)^2}\right)}{2 \times 9.81} + 10 + \frac{\left(\frac{0.25}{\frac{\pi}{4}(0.3)^2}\right)^2}{9.81}$$

$$\boxed{h_p = 16.1\, m}$$

$$\dot{W}_p = \gamma Q h_p$$
$$= 9810 \times (0.25) \times (16.1) = 39485.25 \text{ watt}$$

Subject Date No.

** sudden expansion:- (turbelant flow $\alpha_1 \& \alpha_2 = 1$)



① $P_1A_2 - P_2A_2 - \rho g \Delta L A_2 \sin\alpha$
$= V_2 \rho A_2 V_2 - V_1 (\rho A V_1)$

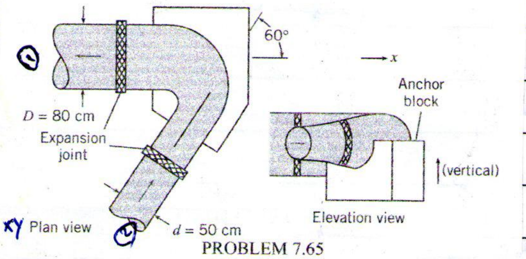
② $\frac{P_1}{\gamma_1} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma_2} + Z_2 + \frac{V_2^2}{2g} + h_L$

③ $V_1A_1 = V_2A_2$

$\Rightarrow \boxed{h_L = \frac{(V_1 - V_2)^2}{2g}}$

Subject Date No.

**7.65** Water flows in this bend at a rate of 5 $m^3/s$, and the pressure at the inlet is 650 kPa. If the head loss in the bend is 10 m, what will the pressure be at the outlet of the bend? Also estimate the force of the anchor block on the bend in the $x$ direction required to hold the bend in place. Assume $\alpha = 1.0$ at all locations.




PROBLEM 7.65

$Q = 5\ m^3/s$

$P_1 = 650\ kPa$

$h_L = 10\ m$

$D_1 = 80\ cm$

$D_2 = 50\ cm$

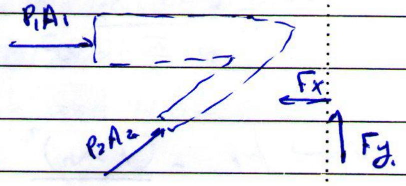
$$V_1 = \frac{Q}{A_1} = \frac{5}{\frac{\pi}{4}(0{,}8)^2} = 9{,}95\ m/s$$

$$V_2 = \frac{Q}{A_2} = \frac{5}{\frac{\pi}{4}(0{,}5)^2} = 25{,}5\ m/s$$

$$\frac{P_1}{\gamma} + Z_1 + \frac{V_1^2}{2g} = \frac{P_2}{\gamma} + Z_2 + \frac{V_2^2}{2g} + h_L$$

$$\frac{650}{9{,}81} + \frac{9{,}99}{2*9{,}81} = \frac{P_2}{9{,}81} + \frac{(25{,}5)^2}{2*9{,}81} + 10$$

$$\boxed{P_2 = 826{,}7\ kPa}$$




$$P_1A_1 - F_x + P_2A_2\cos 60 = (-V_2\cos 60\,(\rho Q)) - V_1(\rho Q)$$

$$F_x = P_1A_1 + P_2A_2\cos 60 + V_2\cos 60\,\rho Q + V_1\rho Q$$

$$= 650*10^3 * \frac{\pi}{4} * (0{,}8)^2 + 826{,}7*10^3\ \frac{\pi}{4}(0{,}5)^2\cos 60$$

$$+ 25{,}5\cos 60\,(5000) + 9{,}95\,(5000)$$

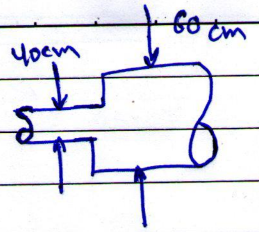
$$= 592740\ N$$

$$= 592{,}740\ kN$$

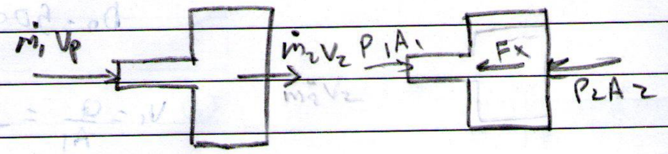
N O T E B O O K

Subject Date No.

**7.61** A 40 cm pipe abruptly expands to a 60 cm size. These pipes are horizontal, and the discharge of water from the smaller size to the larger is 1.0 $m^3/s$. What horizontal force is required to hold the transition in place if the pressure in the 40 cm pipe is 70 kPa gage? Also, what is the head loss? Assume $\alpha = 1.0$ at all locations.




+ احسب القوة التي تجعل الانبوبة مكانها ؟




$$V_1 = \frac{Q}{A_1} = \frac{0.8}{\frac{\pi}{4}(0.4)^2} = 6.36 \text{ m/s}$$

$$V_2 = \frac{Q}{A_2} = \frac{0.8}{\frac{\pi}{4}(0.6)^2} = 2.83 \text{ m/s}$$

بسبب التوسع الفجائي $h_L = \frac{(V_1 - V_2)^2}{2g} = \frac{(6.36 - 2.8)^2}{19.62} = 0.64 \text{ m}$

* يوجد خسارة ودفعان نطبق Energy Eqn

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{z_1} + \cancel{h_p} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{z_2} + \cancel{h_t} + h_L$$

$$\frac{P_2}{\gamma} = \frac{P_1}{\gamma} + \frac{V_1^2 - V_2^2}{2g} - h_L$$

$$\boxed{P_2 = 79.951 \text{ kpa}}$$

$$\sum F_x = \frac{d}{dt}\int_{c.v} \rho V_x dx + \sum \dot{m}_o V_o - \sum \dot{m}_i V_i$$

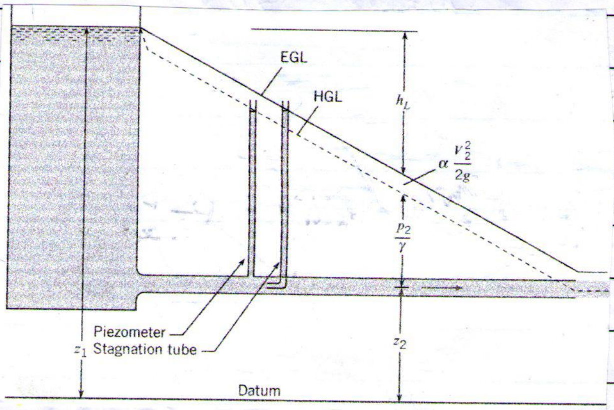
$$P_1 A - P_2 A - F_x = \dot{m}(V_2 - V_1)$$

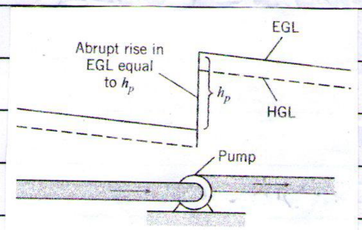
$$\Rightarrow \boxed{F_x = -10.985}$$

$= 10.985 \rightarrow$

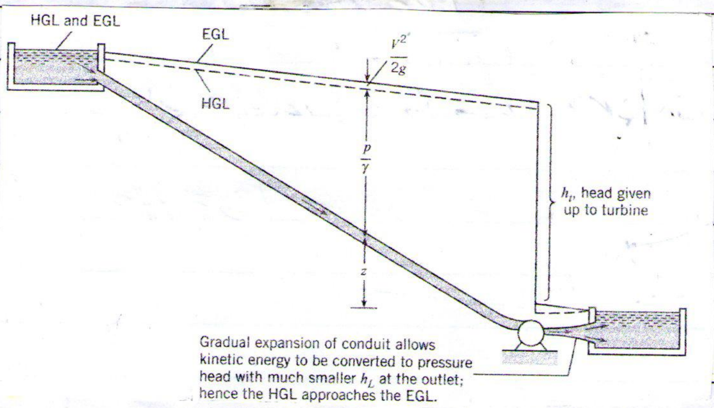
Subject Date No.

## ** hydraulic and energy grid Line :-

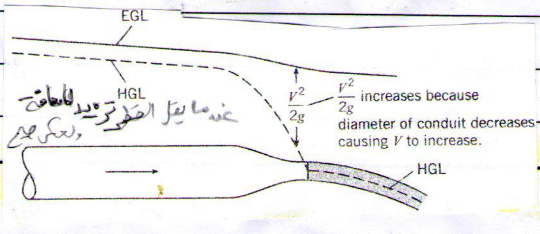



an exception to this will be in the case of pump which will cause an abrupt increase in EGL and HGL




Energy is taken from the system by a turbine on abrupt drop in EGL and HGL will be noticed



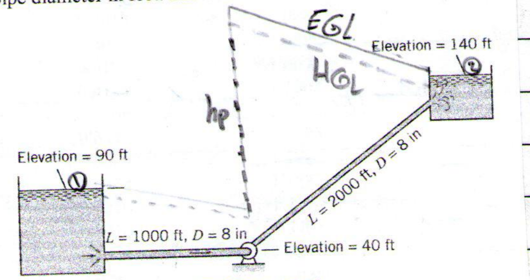

chang in size of pipe :-

(A) an expressed through the space between EGL and HGL

(B) also through the slope

Subject  Date  No.

**7.80** What horsepower must be supplied to the water to pump 3.0 cfs at 68°F from the lower to the upper reservoir? Assume that the head loss in the pipes is given by $h_L = 0.018\,(L/D)\,(V^2/2g)$, where $L$ is the length of the pipe in feet and $D$ is the pipe diameter in feet. Sketch the HGL and the EGL.




PROBLEM 7.80

$Q = 3\,cfs$

$h_L = 0.018\,\frac{L}{D}\,\frac{V^2}{2g}$

$\dot{w}_p = ??$

$$\cancel{\frac{P_1}{\gamma}} + Z_1 + \cancel{\frac{V_1^2}{2g}} + h_p = \cancel{\frac{P_2}{\gamma}} + \cancel{\frac{\alpha V_2^2}{2g}} + Z_2 + H_L$$

$$h_p = h_L + (Z_2 - Z_1)$$

$$= 0.018\,\frac{L}{D}\,\frac{V^2}{2g} + (Z_2 - Z_1)$$

$$V = \frac{Q}{A} = \frac{3}{\frac{\pi}{4}\left(\frac{8}{12}\right)^2} = 8.59\ ft/s$$

$$h_p = 0.018 \times \frac{3000}{\left(\frac{8}{12}\right)} \times \frac{(8.59)^2}{2 \times 32.2} + (140 - 90)$$

$$\boxed{h_p = 128.5\ ft}$$

$$\dot{w}_p = \gamma Q h_p = 62.4 \times 3 \times 128.5 = 24055\ lbft/s$$

$$\div 550 = 43.7\ Hp$$

hourse power

$$\boxed{SI \xrightarrow{\div 746} Hp}$$

N O T E B O O K

Subject **Ch10: Flow in conduits** Date No.

ای جری

* Flow → Laminar — صفائحي
  → turbulent — مضطرب (تداخل الجزيئات)

$Re < 2000$ , $Re > 3000$ , $2000 \leq Re \leq 3000$

Laminar — Turbulent — transiant انتقالي

* Reynolds number:

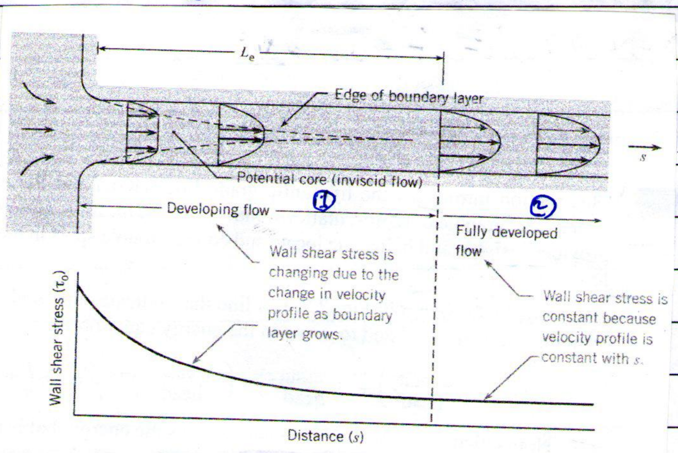
$$Re = \frac{\rho V D}{\mu} \Rightarrow \frac{\rho V L}{\mu}$$ الطول المميز

$$= \frac{VD}{\nu} = \frac{\pi D}{\frac{\pi}{4} D^2 \nu} = \frac{4Q}{\pi D \nu}$$

نيو — $\nu$

$$= \frac{4 \dot{m}}{\pi D \mu}$$

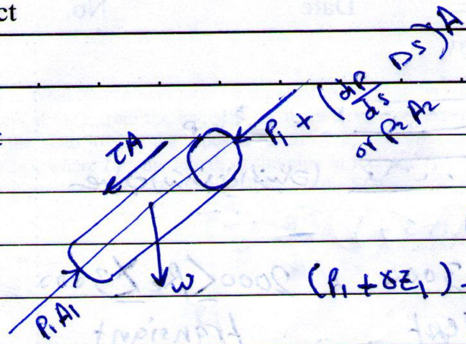
** developing flow ① and Fulling developed flow ②




$\frac{L_e}{D} = 0.05 Re \Rightarrow$ laminar

$\frac{L_e}{D} = 50 \Rightarrow$ Turbulent

***




$$\Sigma F_x = \Sigma V \dot{m} = 0$$

$$(P_1 - P_2)A - W \sin\alpha - \tau(2\pi r \cdot L) = 0$$

$$(P_1 + \gamma Z_1) - (P_2 - \gamma Z_2) = \frac{4\tau_o \Delta L}{D}$$

energy equ.:- $\left(\frac{P_1}{\gamma} + Z_1 + \cancel{\frac{V_1^2}{2g}} = \frac{P_2}{\gamma} + Z_2 + \cancel{\frac{V_2^2}{2g}} + h_L\right)$

$V_1 = V_2$

$$(P_1 + \gamma Z_1) - (P_2 + \gamma Z_2) = \gamma h_L$$

$$\gamma h_L = \frac{4\tau_o \Delta L}{D}$$

$$\boxed{h_L = \frac{4\tau_o \Delta L}{\gamma D}}$$

$$h_L = \frac{L}{D}\left(\frac{4\tau_o}{\frac{1}{2}\rho V_o^2}\right)\left(\frac{\frac{1}{2}\rho V_o^2}{\gamma}\right)$$

dynamic head

$\rho g$

(جزء من $h_L$ ناتج من تغير الماء او المرور الصاح) ناتج سطوح ال pipe $\boxed{h_L = F \times \frac{L}{D} \times \frac{V^2}{2g}}$

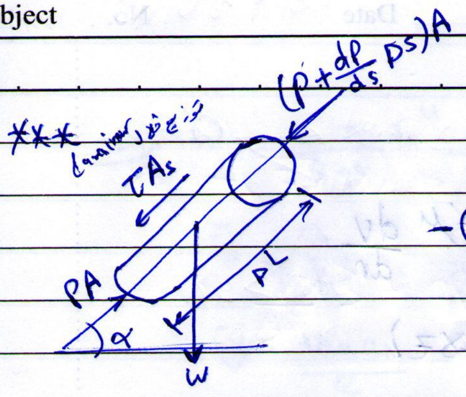
Friction Factor $\Leftarrow$ $\boxed{F = \frac{4\tau_o}{\frac{1}{2}\rho V^2}}$

, Darcy Friction Factor

$$F_{darcy} = 4 F_{fanning}$$

$$\frac{4\tau_o}{\frac{1}{2}\rho V^2}$$

Subject Date No.




$$\sin\alpha = \frac{\Delta z}{\Delta s} = \frac{dz}{ds}$$

$$-\left(\frac{dp}{ds}\cancel{\Delta s}\right)A - \tau(2\pi r\cancel{\Delta s})$$

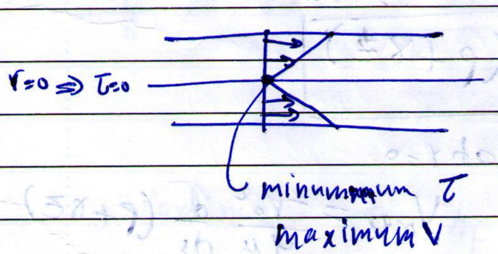
$$-\gamma\cancel{\Delta s A}\frac{dz}{ds} = 0$$

$$-\left(\frac{dp}{ds}\right) - \frac{\tau(2\cancel{\pi}\cancel{r})}{\cancel{\pi}r^2} - \gamma\frac{dz}{ds} = 0$$

$$-\frac{dp}{ds} - \frac{2\tau}{r} - \gamma\frac{dz}{ds} = 0$$

$$-\frac{d}{ds}(p+\gamma z) = \frac{2\tau}{r}$$

$$\tau = \frac{-r}{2}\frac{d}{ds}(p+\gamma z)$$

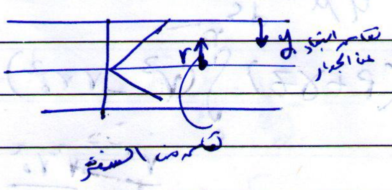



at $r = r_0$

$$\tau_0 = \frac{-r_0}{2}\frac{d}{ds}(p+\gamma z)$$

$\Longrightarrow$ For laminar $\tau = \mu\frac{dv}{dy}$

$$\frac{dv}{dy} = \frac{-dv}{dr}$$




$$\tau = -\mu\frac{dv}{dr}$$

$\Longrightarrow$

Subject Date No.

$$\Rightarrow \tau = -\mu \frac{dv}{dr}$$

$$-\frac{r}{2}\frac{d}{ds}(p+\gamma z) = -\mu \frac{dv}{dr}$$

$$\frac{dv}{dr} = \frac{r}{2\mu}\frac{d}{ds}(p+\gamma z)$$

$$v(r) = \frac{r^2}{2\mu}\frac{d}{ds}(p+\gamma z) + C_1$$

at $r = r_0 \Rightarrow v = 0$

$$0 = \frac{r_0^2}{4\mu}\frac{d}{ds}(p+\gamma z) + C_1$$

$$C_1 = \frac{-r_0^2}{4\mu}\frac{d}{ds}(p+\gamma z)$$

$$\boxed{v(r) = \frac{r^2 - r_0^2}{4\mu}\frac{d}{ds}(p+\gamma z)}$$

at $r = 0$

$$V_{max} = \frac{-r_0^2}{4\mu}\frac{d}{ds}(p+\gamma z)$$

$$Q = \int v\,dA = \int_0^{r_0} \frac{r^2 - r_0^2}{4\mu}\frac{d}{ds}(p+\gamma z)(2\pi r\,dr)$$

$$= \frac{\pi}{2\mu}\frac{d}{ds}(p+\gamma z)\int_0^{r_0}(r^3 - r r_0^2)\,dr$$

$$\frac{r^4}{4} - \frac{r^2 r_0^2}{2}$$

$$\frac{r_0^4}{4} - \frac{r_0^2}{2}$$

$$-\frac{r_0^2}{4}$$

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$$\Rightarrow Q = \frac{-\pi r_0^4}{8\mu} \frac{d}{ds}(p+\gamma z)$$

$$\bar{V} = \frac{Q}{A} = \frac{-r_0^2}{8\mu} \frac{d}{ds}(p+\gamma z)$$

$$\bar{V} = \frac{V_{max}}{2}$$

$$\bar{V} = \frac{-D_0^2}{32\mu} \frac{d}{ds}(p+\gamma z) \qquad \boxed{\alpha_{\text{laminar flow}} = 2}$$

$$\Rightarrow \frac{d}{ds}(p+\gamma z) = 32\mu\bar{V}$$

$$d(p+\gamma z) = -\frac{32\mu\bar{V}}{D^2} ds$$

$$(p_2+\gamma z_2) - (p_1+\gamma z_1) = -(s_2-s_1) \frac{32\mu\bar{V}}{D^2}$$

$$\frac{p_2}{\gamma} + z_2 + \frac{32\mu\bar{V}L}{\gamma D^2} = \frac{p_1}{\gamma} + z_1$$

$$\frac{p_1}{\gamma} + z_1 = \frac{p_2}{\gamma} + z_2 + h_L$$

laminar $\Leftarrow h_f = \dfrac{32\mu\bar{V}L}{\gamma D^2}$

$$h_L = h_f + h_{other}$$

laminar and other $\Leftarrow h_f = f \dfrac{L}{D} \dfrac{V^2}{2g}$

$$\Rightarrow f_{laminar} = \frac{64}{Re} \qquad Re = \frac{\rho V D}{\mu} = \frac{VD}{\nu}$$

NOTEBOOK

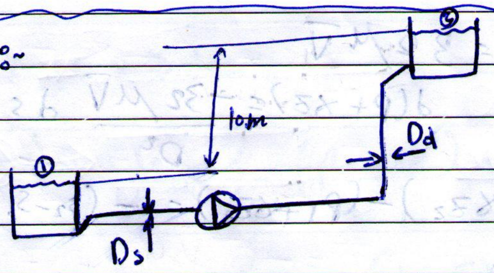
⟹ For turbulent

$$\frac{1}{\sqrt{F}} = 2\log\left(Re\sqrt{F}\right) - 0.8$$

other loses ⟸ $\boxed{h_L = h_f + \sum \frac{k v^2}{2g}}$

[illegible] ** Moody chart & ** table 10.5 (K loses cofficent)

** Type I :-




$Q = 0.015\ m^3/s$

$D_s = 4in$

$D_d = 2in$

$k_s = 4.6 \times 10^{-5}\ m$

$s.g = 0.789$

$\mu = 5.6 \times 10^{-4}$

الخط قبل المضخة ⟸ suction line = 15m

الخط بعد المضخة ⟸ discharge line = 200m

$\left(\sum k \frac{v^2}{2g}\right)$ ⟸ other loses = 26.05 m

power pump = ???

⟹

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Sol:- * $\cancel{\frac{P_1}{\gamma}} + Z_1 + \cancel{\frac{V_1^2}{2g}} + h_P = \cancel{\frac{P_2}{\gamma}} + Z_2 + \cancel{\frac{V_2^2}{2g}} + h_L$

$h_P = h_L + (Z_2 - Z_1)$

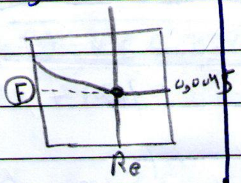
$h_P = (h_f + h_{other}) + (Z_2 - Z_1)$

$= (h_f + h_{f_d} + 26{,}05) + 10$

$= h_{f_s} + h_{f_d} + 36{,}05\ m$

* suction line $\Rightarrow h_{f_s} = f_s \frac{L_s}{D_s} \frac{V_s^2}{2g}$

From moody chart $F_s = 0{,}018$




$h_{f_s} = 0{,}018 \times \frac{15}{4 \times 0{,}0254} \times \frac{(1{,}83)^2}{2 \times 9{,}81}$

$= 0{,}45\ m$

$V_s = \frac{Q}{\frac{\pi}{4}(D_s)^2} = 1{,}83\ m/s$

$Re = \frac{\rho V D}{\mu} = \frac{789 \times 1{,}83 \times 4 \times 0{,}0254}{56 \times 10^{-4}}$

$= 2{,}64 \times 10^5$

$\frac{K_s}{D} = \frac{4{,}6 \times 10^{-5}}{4 \times 0{,}0254}$

$= 0{,}00045$

* discharge line $\Rightarrow$

$V_d = \frac{Q}{A} = \frac{0{,}015}{\frac{\pi}{4}(2 \times 0{,}254)^2}$

$= 6{,}92\ m/s$

$Re = \frac{789 \times 6{,}92 \times 2 \times 0{,}0254}{5{,}6 \times 10^{-4}} = 5{,}12 \times 10^5$

$\frac{K_s}{D} = \frac{4{,}6 \times 10^{-5}}{2 \times 0{,}0254} = 0{,}0009$

From Moody chart $F_d = 0{,}02$

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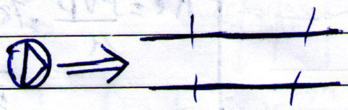
$\Longrightarrow h_{fd} = 0,02 * \frac{200}{2*0,025} * \frac{(6,92)^2}{2*9,81} = 185,9 \text{ m}$

$h_p = h_{fs} + h_{fd} + 36,05 = 0,49 + 185,9 + 36,05$
$= 222,4$

$*$ power pump $= \gamma Q h_p = 7,81 * 789 * 0,015 * 222,4$
$= 25,82 \text{ kW}$

** Type II :-



$D = 6 \text{in}$

$\frac{\Delta P}{100\text{m}} = 60 \text{ kPa}$

$\rho = 880 \text{ kg/m}^3$

$\mu = 9,5 * 10^5 \text{ Pa.s}$

$K = 4,6 * 10^{-5}$

Sol:-

$h_L = h_f$

$h_L = \frac{P_1 - P_2}{\gamma} = \frac{60*1000}{9,81*880} = 6,95 \text{m}$

$h_f = f \frac{L}{D} \frac{V^2}{2g} \Longrightarrow V = \sqrt{\frac{2g\, h_f}{F} * \frac{D}{L}}$

$Re\sqrt{F} = \frac{D^{3/2}}{\nu} \left(\frac{2 g\, h_f}{L}\right)^{\frac{1}{2}} \Longrightarrow$

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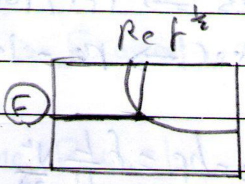
$$Re\sqrt{F} = \frac{(6 \times 0{,}0254)^{3/2}}{\frac{9{,}5 \times 10^{-6}}{880}} \left( \frac{2 \times 9{,}81 \times 6{,}45}{100} \right)^{1/2} = 8805{,}94$$

$$\frac{k_s}{D} = \frac{4{,}6 \times 10^{-6}}{6 \times 0{,}0254} = 0{,}0003$$




$$\underline{F = 0{,}0225}$$

$$V = \sqrt{\frac{2 \times 9{,}81 \times 6{,}95 \times 6 \times 0{,}0254}{0{,}0225 \times 100}} = \underline{3{,}06 \ m/s}$$

اقصى سرعة

$$Q = VA = 3{,}06 \times \frac{\pi}{4} (6 \times 0{,}0254)^2 = 0{,}05581$$

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** Type III :- find the size :-

* comercial steel pipe $\Rightarrow K_s = 1{,}5 * 10^{-4}$ ft
* volumetre flow rate $\Rightarrow$ 300 cfs
* loses $\Rightarrow$ 1 ft / 1000 ft
* water $\Rightarrow \nu = 1{,}22 * 10^{-5}$ ft²/s

sol)

$$h_L = h_f = f \frac{L}{D} \frac{V^2}{2g} = F \frac{L}{D} \frac{Q^2/A^2}{2g} = F \frac{L}{D} \frac{(Q^2 * 16)}{\pi^2 D^4 (2g)}$$

$$= F \frac{* 1000 * 16 * (300)^2}{2g \pi^2 D^5} \quad C$$

$$1 = C \frac{F}{D^5} \Rightarrow D = C^{\frac{1}{5}} F^{\frac{1}{5}}$$

$D = 18{,}665\, F^{1/5} \Longrightarrow$ assume $F = 0{,}02$

$D = 8{,}535$ ft

$$V = \frac{Q}{A} = \frac{300}{\frac{\pi}{4}(8{,}535)^2} = 5{,}243$$

$$Re = \frac{VD}{\nu} = \frac{5{,}243 * 8{,}535}{1{,}22 * 10^{-5}} = 3{,}67 * 10^{6}$$

$$\frac{k_s}{D} = \frac{1{,}5 * 10^{-4}}{8{,}535} = 0{,}000018$$

From moody chart $\Rightarrow F = 0{,}012$

* خطوات الحل :
1- يتم فرض F
2- نحسب D
3- $V = \frac{Q}{A}$
4- Re
5- $\frac{k_s}{D}$
6- F
7- تقارن F مع assume F

الى ان تتساوى F او تكون قريبة منها

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⟹ 2 itration ⟶ assume $F = 0{,}012$

$D = 18{,}66 * (0{,}012)^{1/5} = 7{,}7\ ft$

$\frac{K_s}{D} = 0{,}000019$

$Re = 4{,}062 * 10^6$

⟹ $F = 0{,}01$

⟹ 3 itration ⟶ assume $F = 0{,}01$

$D = 7{,}43\ ft$

$\frac{K_s}{D} = 2{,}01 * 10^{-5}$

$Re = 4{,}213 * 10^6$

⟹ $F = 0{,}01$

So $D = 7{,}43 * 12 = 89{,}16\ in$

From size table standard

$D = 90\ in$

تاخذ الأكبر

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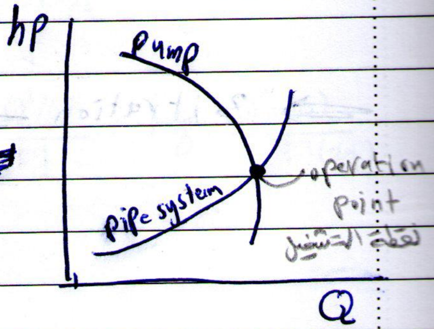
*** operating point and pump power :-

$$\text{pump power} = \gamma Q h_p$$

* حتى يرفع الـ flow لمسافات عالية
يحتاج Q أقل و hp أقل

* كاتالوج المضخة




$$h_p = \frac{P_2 - P_1}{\gamma} + Z_2 - Z_1 + h_L + \frac{V_2^2 - V_1^2}{2g}$$

$$V_1 = \frac{Q}{A_1} \qquad V_2 = \frac{Q}{A_2}$$

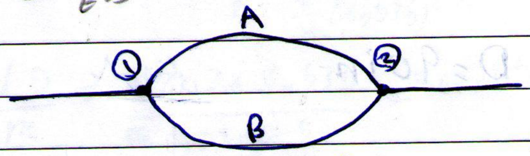
$$h_L = \sum K \frac{V^2}{2g} + h_f$$

$$\left(F \frac{L}{D} \frac{V^2}{2g}\right)$$

* pipe V تزداد ⇐ h تزداد
Q تزداد ⇐ h تزداد

* pump علاقة عكسية

** إذا كان التوزيع على شبكة **




$$h_{L_1} = h_{L_2}$$

$$\left(F \frac{L}{D} \frac{V^2}{2g}\right)_A = \left(F \frac{L}{D} \frac{V^2}{2g}\right)_B$$

$$Q = Q_A + Q_B$$

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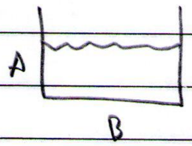
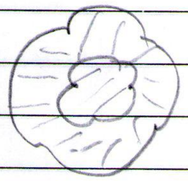
** $D_{hydroulic} = \dfrac{4A}{w_{dia}}$

(القطع ليس دائري)

الطول المبلل




$w = 2A + B$



محيط الصغيرة + محيط الكبيرة .

For check

$$\boxed{D_{\text{دائرة}} = 4\left(\frac{\frac{\pi}{4} D^2}{\pi D}\right) = \underline{\underline{D}}}$$